Circuit Theory I
Basic Concepts

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AnantAgarwal and Jeffrey Lang, course materials for 6.002 Circuits and Electronics, Spring 2007. MIT OpenCourseWare(http://ocw.mit.edu/), Massachusetts Institute of Technology
Electrical engineering

What is engineering?

The application of scientific, economic, social, and practical knowledge in order to design, build, maintain, and improve structures, machines, devices, systems, materials and processes (Source: wikipedia).

Purposeful use of science (cite: Steve Sentura).
What is this course about?

**Circuit theory**

- Gainful employment of Maxwell’s equations...
- From electrons to digital gates and op-amps.
- Make it simple though!
Six basic SI units and one derived unit relevant to this course...

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Basic Unit</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Length</td>
<td>meter</td>
<td>m</td>
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<tr>
<td>Mass</td>
<td>kilogram</td>
<td>kg</td>
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<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Electric current</td>
<td>ampere</td>
<td>A</td>
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<tr>
<td>Thermodynamic Temperature</td>
<td>kelvin</td>
<td>K</td>
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<tr>
<td>Luminous intensity</td>
<td>candela</td>
<td>cd</td>
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<tr>
<td>Charge</td>
<td>coulomb</td>
<td>C</td>
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### SI prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Multiplying factor</th>
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From the facts of nature to build interesting systems...

Nature as observed in experiments

tables of data from the measurements&observations

Physics laws

Maxwell’s equations, Ohm’s Law → abstraction for
tables of data

Lumped Circuit abstraction

Simple amplifier abstraction
## Simple amplifier

<table>
<thead>
<tr>
<th>Operational amplifier</th>
<th>Digital abstraction</th>
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<th>Combinational logic</th>
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### Analog system components:
- modulators,
- oscillators,
- RF amps,
- Power supplies

<table>
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<tr>
<th>Clocked digital</th>
<th>Instruction set</th>
<th>Programming languages</th>
<th>Software systems, operating systems, browsers</th>
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Toasters, sonar, stereos, space shuttle, doomsday devices 😊
Electrical charge

- Charge is a characteristic of a unit of matter that expresses the extent to which it has more or fewer electrons than protons.
- Measured in coulombs [C]

- 1 C of charge requires $6.24 \times 10^{18}$ electrons.
- 1 electron charge $e = -1.602 \times 10^{-19}$ C.
- **Law of Conservation of Charge:** Charge can only be transferred. Cannot be created/destroyed.
• Electric currents are flows of electric charge.

• Suppose a collection of charges is moving perpendicular to a surface of area $A$

• The electric current is defined to be the rate at which charges flow across any cross-sectional area. If an amount of charge $\Delta Q$ passes through a surface in a time interval $\Delta t$, then the average current $I_{\text{avg}}$ is given by

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$$
Electrical current

- The SI unit of current is the ampere (A), with 1 A = 1 coulomb/sec.
- Common currents range from mega-amperes in lightning to nano-amperes in your nerves.
- In the limit $\Delta t \to 0$ the instantaneous current $I$ may be defined as

$$I_{ave} = \frac{dq}{dt}$$
Direct current (DC)

- **Direct current (dc)** is the unidirectional flow of electric charge.
Alternating current (AC)

- In *alternating current (ac)*, the flow of electric charge periodically reverses direction
Problems:

Ex. 1.1: How much charge is represented by 4,600 electrons?

Solution:
Each electron has $-1.602 \times 10^{-19}$ C. Hence 4,600 electrons will have
$$-1.602 \times 10^{-19} \text{ C/electron} \times 4,600 \text{ electrons} = -7.369 \times 10^{-16} \text{ C}$$

Ex. 1.2: The total charge entering a terminal is given by $q = 5t \sin 4\pi t \text{ mC}$. Calculate the current at $t = 0.5 \text{ s}$.

Solution:
$$i = \frac{dq}{dt} = \frac{d}{dt} (5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$
At $t = 0.5$,
$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$
Ex. 1.3: Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{2} i \, dt = \int_{1}^{2} (3t^2 - t) \, dt$$

$$= \left. \left( t^3 - \frac{t^2}{2} \right) \right|_1^2 = (8 - 2) - \left( 1 - \frac{1}{2} \right) = 5.5 \, \text{C}$$
Voltage and potential

- **Electrostatic Potential Energy:**
  the work that I have to do to bring a charge to a point $P$

  unit: joules

- **Electric potential, $V$:**
  Energy required to move a unit charge through an element
  unit: volt
Voltage and potential

• Electric Potential, V:
  Electrical location of a point according to a reference.
  unit: volt

• Voltage, U:
  Energy required to move a unit charge through an element
  unit: volt

\[ V_{21} = V_2 - V_1 \]
Potential, $V$:
Electrical location of a point according to a reference.
unit: volt
Voltage, $v_{ab}$: potential difference

unit: volt

$$v_{ab} = \frac{dw}{dq}$$

$v$: voltage in volts (V)
$w$: Energy in joules (J)
$q$: charge in coulombs (C)

$V_{21} = V_2 - V_1$
Voltage

- Voltage pushes charge in one direction.
- We use polarity (+ and −) on batteries to indicate which direction the charge is being pushed.

Two equivalent representation of the same voltage: (a) point \( a \) is 9 V above point \( b \), (b) point \( b \) is −9 V above point \( a \).

- dc voltage is represented by V and ac voltage is represented by \( \nu \).
**Alternating voltage**

Voltage, $U(t)$

$U_e = \text{effective value} = \frac{U_{\text{max}}}{\sqrt{2}}$

Period, $T = \frac{1}{f}$
Direct voltage

\[ u(t) \]

\[ U = +U_o = \text{constant} \]

0

\[ T_1 = 1.67 T \]
\[ T' = 0.3 \quad T_1 = 0.5 \ T \]

IMPULSE VOLTAGE
Lumped circuit abstraction

Consider this circuit:

Suppose we wish to answer this question:

What is the current through the bulb?
Apply Maxwell’s equations...

Faraday’s \[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \oint E \cdot dl = -\frac{\partial \phi_B}{\partial t} \]

Continuity \[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} \]
\[ \oint J \cdot dS = -\frac{\partial q}{\partial t} \]

Others \[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]
\[ \oint E \cdot dS = \frac{q}{\varepsilon_0} \]

It is hard
Instead there is an easy way...

• First let’s build some insight:

\[ F \to m \to a ? \]

• I ask you: what is the acceleration?

• You should ask me: what is the mass?

• I tell you: \( m \)

• You respond: \( a = \frac{F}{m} \)

—Done!
In doing so, we ignored
- The shape of the object
- Its temperature
- Its color
- Point of force application
• Consider the filament of the light bulb.

• We do not care about
  – How current flows inside the filament
  – Its temperature, shape, orientation, etc.

Then we can replace the bulb with a «discrete resistor» for the purpose of calculating the current.
The easy way...

- R represents the only property of interest!

- Like with the point-mass: replace objects with their mass m to find a.
The easy way...

- $R$ represents the only property of interest!

- $R$ relates element $V$ and $I$

\[ I = \frac{V}{R} \]

is called element v-i relationship.
R is a lumped element abstraction for the bulb.
R is a lumped element abstraction for the bulb

- Although we take the easy way using lumped abstractions for the rest of this course, we must make sure (at least the first time) that our abstraction is reasonable.

- In this case, ensure V and I are defined for the element.
What does it buy us?

- Replace differential equations with simple algebra using lumped circuit abstraction.

under some constraints...

\[
\begin{align*}
\frac{\partial \phi_B}{\partial t} &= 0 \quad \text{outside} \\
\frac{\partial q}{\partial t} &= 0 \quad \text{inside elements} \\
&\quad \text{bulb, wire, battery}
\end{align*}
\]

(you will learn them later, keep those in mind...)
• **Power** is the rate of expending and absorbing energy, measured in watts (W).

\[
p = \frac{dw}{dt} = v \cdot i
\]

- \( p \) : Power in watts (W)
- \( v \) : Voltage in volts (V)
- \( w \) : Energy in joules (J)
- \( t \) : Time in seconds (s)
- \( i \) : Current in amperes (A)

- Circuit elements that *absorb* power have *positive* value of \( p \).
- Circuit elements that *supply* (produce) power have *negative* value of \( p \).

(a) Absorbing power (b) supplying power.
• **Energy** is the capacity to do work, measured in joules (J).

\[ w = \int_{t_0}^{t} p \, dt = \int_{t_0}^{t} \pm v \cdot i \, dt \]

- \( p \) : Power in watts (W)
- \( v \) : Voltage in volts (V)
- \( w \) : Energy in joules (J)
- \( t \) : Time in seconds (s)
- \( i \) : Current in amperes (A)

• If current and voltage are constant (dc), the power is constant. The energy is:
\[ w = \int_{t_0}^{t} p \, dt = p(t - t_0) \]

• In addition to joules, Watt-hour can also be used to measure energy.
\[ 1 \text{ Wh} = 3,600 \text{ J} \]

• **Law of conservation of energy.** Total power in a circuit at any instant must be zero.
Ex. 1.4: An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.
Ex. 1.5: Find the power delivered to an element at $t = 3 \text{ ms}$ if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3 \frac{di}{dt}$. 
Ex. 1.5:  (b) We find the voltage and the power as

\[ v = 3 \frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V} \]

\[ p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W} \]

At \( t = 3 \text{ ms} \),

\[ p = -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \]

\[ = -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \]
Ex. 1.5: How much energy does a 100-W electric bulb consume in two hours?
**Passive sign convention** is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

**Ex. 1.7: Sign convention is applied.**

\[ p_1 = 20(-5) = -100 \text{ W} \quad \text{Supplied power} \]
\[ p_2 = 12(5) = 60 \text{ W} \quad \text{Absorbed power} \]
\[ p_3 = 8(6) = 48 \text{ W} \quad \text{Absorbed power} \]
\[ p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \quad \text{Supplied power} \]
Circuit elements

- Ideal independent sources: provides a specified voltage or current that is completely independent of other circuit variables

- Ideal Independent Voltage Source:
  (a) Independent voltage source (constant / time varying)
  (b) Independent voltage source (battery).

- Ideal Independent Current Source:
Circuit elements

- Ideal Dependent Sources: controlled by other voltage or current

(a) dependent voltage source
(b) dependent current source