Sample questions for Power Factor Improvement

EENG456
Example   An alternator is supplying a load of 300 kW at a p.f. of 0.6 lagging. If the power factor is raised to unity, how many more kilowatts can alternator supply for the same kVA loading?

Solution:

\[
\text{kVA} = \frac{\text{kW}}{\cos \phi} = \frac{300}{0.6} = 500 \text{ kVA}
\]

- kW at 0.6 p.f. = 300 kW
- kW at 1 p.f. = 500 \times 1 = 500 kW

\[
\therefore \text{Increased power supplied by the alternator} = 500 - 300 = 200 \text{ kW}
\]

Note the importance of power factor improvement. When the p.f. of the alternator is unity, the 500 kVA are also 500 kW and the engine driving the alternator has to be capable of developing this power together with the losses in the alternator. But when the power factor of the load is 0.6, the power is only 300 kW. Therefore, the engine is developing only 300 kW, though the alternator is supplying its rated output of 500 kVA.
Example  A single phase a.c. generator supplies the following loads:

(i) Lighting load of 20 kW at unity power factor.
(ii) Induction motor load of 100 kW at p.f. 0.707 lagging.
(iii) Synchronous motor load of 50 kW at p.f. 0.9 leading.

Calculate the total kW and kVA delivered by the generator and the power factor at which it works.

Solution: Using the suffixes 1, 2 and 3 to indicate the different loads, we have,

\[
\begin{align*}
\text{kVA}_1 &= \frac{\text{kW}_1}{\cos \phi_1} = \frac{20}{1} = 20 \text{ kVA} \\
\text{kVA}_2 &= \frac{\text{kW}_2}{\cos \phi_2} = \frac{100}{0.707} = 141.4 \text{ kVA} \\
\text{kVA}_3 &= \frac{\text{kW}_3}{\cos \phi_3} = \frac{50}{0.9} = 55.6 \text{ kVA}
\end{align*}
\]

These loads are represented in Fig. The three kVAs’ are not in phase. In order to find the total kVA, we resolve each kVA into rectangular components – kW and kVAR as shown in Fig. The total kW and kVAR may then be combined to obtain total kVA.
\[
\begin{align*}
kVAR_1 &= kVA_1 \sin \phi_1 = 20 \times 0 = 0 \\
kVAR_2 &= kVA_2 \sin \phi_2 = -141.4 \times 0.707 = -100 \text{ kVAR} \\
kVAR_3 &= kVA_3 \sin \phi_3 = +55.6 \times 0.436 = +24.3 \text{ kVAR}
\end{align*}
\]

Note that \(kVAR_2\) and \(kVAR_3\) are in opposite directions; \(kVAR_2\) being a lagging while \(kVAR_3\) being a leading \(kVAR\).

Total \(kW\) = \(20 + 100 + 50 = 170 \text{ kW}\)

Total \(kVAR\) = \(0 - 100 + 24.3 = -75.7 \text{ kVAR}\)

Total \(kVA\) = \(\sqrt{(kW)^2 + (kVAR)^2} = \sqrt{(170)^2 + (75.7)^2} = 186 \text{ kVA}\)

Power factor = \(\frac{\text{Total } kW}{\text{Total } kVA} = \frac{170}{186} = 0.914 \text{ lagging}\)

The power factor must be lagging since the resultant \(kVAR\) is lagging.
Example  A 3-phase, 5 kW induction motor has a p.f. of 0.75 lagging. A bank of capacitors is connected in delta across the supply terminals and p.f. raised to 0.9 lagging. Determine the kVAR rating of the capacitors connected in each phase.

Solution:

Original p.f., $\cos \phi_1 = 0.75$ lag  ;  Motor input, $P = 5$ kW

Final p.f., $\cos \phi_2 = 0.9$ lag  ;  Efficiency, $\eta = 100$ % (assumed)

$\phi_1 = \cos^{-1} (0.75) = 41.41^\circ$ ;  $\tan \phi_1 = \tan 41.41^\circ = 0.8819$

$\phi_2 = \cos^{-1} (0.9) = 25.84^\circ$ ;  $\tan \phi_2 = \tan 25.84^\circ = 0.4843$

Leading kVAR taken by the condenser bank

$$= P (\tan \phi_1 - \tan \phi_2)$$

$$= 5 (0.8819 - 0.4843) = 1.99\text{ kVAR}$$

∴ Rating of capacitors connected in each phase

$$= 1.99/3 = 0.663\text{ kVAR}$$
Example  A 3-phase, 50 Hz, 400 V motor develops 100 H.P. (74.6 kW), the power factor being 0.75 lagging and efficiency 93%. A bank of capacitors is connected in delta across the supply terminals and power factor raised to 0.95 lagging. Each of the capacitance units is built of 4 similar 100 V capacitors. Determine the capacitance of each capacitor.

Solution:

Original p.f., \( \cos \phi_1 = 0.75 \) lag; Final p.f., \( \cos \phi_2 = 0.95 \) lag

Motor input, \( P = \frac{\text{output} \times \eta}{\text{output}} = \frac{74.6}{0.93} = 80 \text{ kW} \)

\[ \phi_1 = \cos^{-1} (0.75) = 41.41^\circ \]

\[ \tan \phi_1 = \tan 41.41^\circ = 0.8819 \]

\[ \phi_2 = \cos^{-1} (0.95) = 18.19^\circ \]

\[ \tan \phi_2 = \tan 18.19^\circ = 0.3288 \]

Leading kVAR taken by the condenser bank

\[ = P (\tan \phi_1 - \tan \phi_2) \]
\[ = 80 (0.8819 - 0.3288) = 44.25 \text{ kVAR} \]

Leading kVAR taken by each of three sets

\[ = 44.25/3 = 14.75 \text{ kVAR} \]

\( ... (f) \)
Fig. shows the delta* connected condenser bank. Let $C$ farad be the capacitance of 4 capacitors in each phase.

Phase current of capacitor is

\[
I_{CP} = \frac{V_{ph}}{X_C} = 2\pi f C V_{ph} \\
= 2\pi \times 50 \times C \times 400 \\
= 1,25,600 \ C \text{ amperes}
\]

kVAR/phase = \[
\frac{V_{ph} I_{CP}}{1000} \\
= \frac{400 \times 1,25,600 \ C}{1000} \\
= 50240 \ C \quad \ldots \ (if)
\]

* In practice, capacitors are always connected in delta since the capacitance of the capacitor required is one-third of that required for star connection.
Equating exps. (i) and (ii), we get,

\[ 50240 \ C = 14.75 \]

\[ \therefore \quad C = \frac{14.75}{50240} = 293.4 \times 10^{-6} \ F = 293.4 \ \mu F \]

Since it is the combined capacitance of four equal capacitors joined in series,

\[ \therefore \quad \text{Capacitance of each capacitor} = 4 \times 293.4 = 1173.6 \ \mu F \]
Example  A supply system feeds the following loads (i) a lighting load of 500 kW (ii) a load of 400 kW at a p.f. of 0.707 lagging (iii) a load of 800 kW at a p.f. of 0.8 leading (iv) a load of 500 kW at a p.f. 0.6 lagging (v) a synchronous motor driving a 540 kW d.c. generator and having an overall efficiency of 90%. Calculate the power factor of synchronous motor so that the station power factor may become unity.

Solution. The lighting load works at unity p.f. and, therefore, its lagging kVAR is zero. The lagging kVAR are taken by the loads (ii) and (iv), whereas loads (iii) and (v) take the leading kVAR. For station power factor to be unity, the total lagging kVAR must be neutralised by the total leading kVAR. We know that kVAR = kW tan φ.

\[
\therefore \text{ Total lagging kVAR taken by loads (ii) and (iv)}
\]
\[
= 400 \tan \left( \cos^{-1} 0.707 \right) + 500 \tan \left( \cos^{-1} 0.6 \right)
\]
\[
= 400 \times 1 + 500 \times 1.33 = 1065
\]

Leading kVAR taken by the load (iii)

\[
= 800 \tan \left( \cos^{-1} 0.8 \right) = 800 \times 0.75 = 600
\]

\[
\therefore \text{ Leading kVAR to be taken by synchronous motor}
\]
\[
= 1065 - 600 = 465 \text{ kVAR}
\]

Motor input = output/efficiency = 540/0.9 = 600 kW

If φ is the phase angle of synchronous motor, then,

\[
\tan \phi = \text{kVAR/kW} = 465/600 = 0.775
\]

\[
\therefore \phi = \tan^{-1} 0.775 = 37.77^\circ
\]

\[
\therefore \text{ p.f. of synchronous motor} = \cos \phi = \cos 37.77^\circ = 0.79 \text{ leading}
\]

Therefore, in order that the station power factor may become unity, the synchronous motor should be operated at a p.f. of 0.79 leading.