RECURSIVE INVERSE BASIS FUNCTION (RIBF) ALGORITHM FOR IDENTIFICATION OF PERIODICALLY VARYING SYSTEMS

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ABSTRACT

This paper presents a new algorithm for the identification (tracking) of periodically varying systems. When the system coefficients vary rapidly, conventional adaptive estimators such as the least mean squares (LMS) and the weighted least squares (WLS) algorithms become inefficient. Basis function (BF) algorithms have shown superiority over the conventional ones in tracking the parameters of periodically varying systems. Unfortunately, BF estimators are computationally very demanding. A new recursive inverse basis function estimator (RIBF) and its frequency-adaptive version are proposed which provides a significant reduction in the computational complexity and the mean square parameter estimation error without the need for any error correction code.

***Index Terms*—** Basis function algorithms, system identification, nonstationary process, periodically varying systems, adaptive filters

1. INTRODUCTION

The distortion introduced by the multipath effect is one of the main problems of modern wireless communications. The multipath effect causes interference between two or more versions of the transmitted signal which arrive at the receiver along different paths at slightly different times. When the multipath signal becomes dominant by some strong reflectors and when the transmitter or receiver moves with constant velocity, the channel-impulse response becomes periodic. The multipath effect phenomena can be modelled as a periodically varying system problem stated in the following mathematical relation

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\theta}(t) + \upsilon(t) \tag{1}$$

where $t = 1, 2, \ldots$ denotes normalized discrete time, $\upsilon(t)$ is an additive white noise and $y(t)$ is the system output. $\boldsymbol{\varphi}(t) = [u(t), u(t-1), \ldots, u(t-n+1]^T$ is the regression vector of the past input samples, $\boldsymbol{\theta}(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_n(t)]^T$ is the vector of n periodically time varying system coefficients. The i^{th} system coefficient is modelled as a sum of weighted exponentials as follows

$$\theta_i(t) = \sum_{l=1}^{k} a_{il} e^{j\omega_l t} \tag{2}$$

where k is the number of different signal paths (fading path or number of basis functions). Adaptive filters could be employed efficiently for identifying periodically time-varying systems under certain conditions [1]. Considering that the frequencies $\omega_1, \omega_2, \ldots, \omega_k$ are given or estimated a prior, the parameters or system weighting coefficients a_{il} are unknown constants and independent of time, or (possibly) slowly time-varying quantities. The input sequence is considered as a wide-sense stationary WSS ergodic process with known positive definite covariance matrix $\boldsymbol{\Phi}_0 = E\left[\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^T(t)\right] > 0$.

Under certain conditions one of the applications that admits such formulation is adaptive equalization of rapidly fading communication channels [2]. It is known that in modern wireless communication systems, the main distortion that the transmitted signal experiences during transmission is caused mostly by the multipath effect; the transmitted signal reaches usually a moving receiver (with constant speed) with different time delays. When this effect is dominant through different reflectors, this leads to an unknown periodic channel impulse response which can be modeled by (2). In this particular case $y(t)$ stands for the sampled baseband signal received by the mobile radio system for time sequence $t = 1, 2, \ldots$,. $u(t)$ is the transmitted symbols sequence (which includes the training sequence), k is the number of signal paths, and $\omega_1, \omega_2, \ldots, \omega_k$ are the corresponding Doppler shifts.

When the system parameters change very rapidly, conventional estimators such as WLS and LMS become unsatisfactory. Hence, it would be more advantageous to use a special form of estimators such as the BF algorithms, which focus on the parameter estimation aspect, as shown in (2). However, BF estimators suffer from high computational complexity.

The exponential model described above, which is adapted for mobile radio channels, has been proposed by Aiken in 1967. Recently, the same model has been successfully applied for equalization and used in aeronautical radio and underwater acoustic channels [5], [3].

This paper focuses on the parameter tracking aspects of the complex exponential model and on the computational complexity criteria. It contributes to the area of adaptive equalization and communication fields by proposing a new efficient BF estimator, termed as the recursive inverse basis function (RIBF). Moreover, for a satisfactory equalization process, a frequency-adaptive version of RIBF is developed by means of a simple gradient search strategy. The new BF estimator outperforms the exponentially weighted basis function (EWBF) estimator by providing considerable complexity savings. RIBF is superior to the Gradient-BF and EWBF estimators by further reducing the mean square parameter estimation error without using any correction code. These result in significant advantages when applied to wireless communications to reduce BER, SNR and channel bandwidth requirements.

This paper is organized as follows: in section 2 the existing BF (i.e. GBF and EWBF) estimators are reviewed. In section 3, the new RIBF estimator and its frequency adaptive version are derived. In section 4, computational complexities of the different BF estimators are evaluated. In section 5, simulation results compare the performance of the proposed estimator with those of GBF and EWBF. Finally, in the last part, conclusions are drawn.

2. BASIS FUNCTION ESTIMATORS

The basis function set consist of the basis exponentials $\{f_1(s), f_2(s), \ldots, f_k(s), s \in T_t\}$, where $T_t = [1, 2, \ldots, t]$ is the expanding time analysis window. Each basis element is given by

$$f_l(s) = e^{j\omega_l s}, \qquad l = 1, \ldots, k \tag{3}$$

These are linearly independent in the time frame T_t, if all the frequency components differ, i.e., $\omega_i \neq \omega_l$ for $i \neq l$. The basis function algorithm corresponds to the following forward-time description of coefficient changes [4],

$$\theta_i(s) = \sum_{l=1}^{k} a_{il} f_l(s), \qquad s \in T_t, \quad i = 1, \ldots, n \tag{4}$$

In vector form, by making use of the Kronecker product, (APPENDIX A), (4) can be written as follows

$$\boldsymbol{\theta}(s) = \left(\mathbf{I}_n \otimes \mathbf{f}^T(s)\right)\boldsymbol{\alpha}(s) = \mathbf{D}(s)\boldsymbol{\alpha}(s) \tag{5}$$

where $\boldsymbol{\theta}(s)$ is the vector of periodically time varying system coefficients, $\mathbf{D}(s) = \mathbf{I}_n \otimes \mathbf{f}^T(s)$, $\mathbf{I}_n$ is n-by-n identity matrix, $\boldsymbol{\alpha} = [a_{11}, a_{12}, \ldots, a_{1k}, \ldots, a_{n1}, a_{n2}, \ldots, a_{nk}]^T$ is the nk-by-1 vector of unknown coefficients, and $\mathbf{f}(s) = [f_1(s), f_2(s), \ldots, f_k(s)]^T$ is the vector of basis functions. Using the Kronecker product, a generalized regression vector can be written as

$$\boldsymbol{\psi}(s) = \left[u(s)\mathbf{f}^T(s), u(s-1)\mathbf{f}^T(s), \ldots, u(s-n+1)\mathbf{f}^T(s)\right]^T = \boldsymbol{\varphi}(s) \otimes \mathbf{f}(s)$$

The system equation in (1) can be stated as follows

$$y(s) = \boldsymbol{\psi}^T(s)\boldsymbol{\alpha} + v(s), \quad s \in T_t \tag{6}$$

It is clear that the basis functions method is based on an explicit model of parameter variation. The parameter trajectory is approximated by a linear combination of known time-functions called the basis functions. The generalized regression vector in (6) demonstrates that the problem of identification of a linear time-varying system of order n is converted to a problem of coefficient estimation of a linear time-invariant system of order nk.

2.1. Exponentially Weighted Basis Function (EWBF) Estimator

The method of weighted least squares has the ability to track small variations due to its finite memory property. Using the exponentially weighted least squares method, we can get the following estimate of $\boldsymbol{\alpha}$ [2]:

$$\hat{\boldsymbol{\alpha}}(t) = \arg\min_{\boldsymbol{\alpha}} \sum_{t=0}^{t-1} \lambda^i \left| y(t-i) - \boldsymbol{\psi}^T(t-i)\boldsymbol{\alpha} \right|^2 = \left(\mathbf{R}^*(t)\right)^{-1}\mathbf{s}(t) \tag{7}$$

where

$$\mathbf{R}(t) = \sum_{t=0}^{t-1} \lambda^i \boldsymbol{\psi}(t-i)\boldsymbol{\psi}^H(t-i) = \lambda\mathbf{R}(t-1) + \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t)$$
$$\mathbf{s}(t) = \sum_{t=0}^{t-1} \lambda^i y(t-i)\boldsymbol{\psi}^*(t-i) = \lambda\mathbf{s}(t-1) + y(t)\boldsymbol{\psi}^H(t) \tag{8}$$

where λ is the forgetting factor, usually close and smaller than one, $\mathbf{R}(t)$ is the nk-by-nk input correlation matrix, and $\mathbf{s}(t)$ is the (nk-by-1) cross-correlation vector between the desired response $y(t)$ and the input vector $\boldsymbol{\psi}(t)$.

Applying the matrix inversion lemma to (8), one arrives to the EWBF algorithm (9) [5], which recursively estimates $\hat{\boldsymbol{\theta}}$.

$$\begin{aligned}
&\hat{\boldsymbol{\theta}} = \mathbf{D}(t)\hat{\boldsymbol{\alpha}}(t)\\
&\hat{\boldsymbol{\alpha}}(t) = \hat{\boldsymbol{\alpha}}(t-1) + \boldsymbol{K}^*(t)\varepsilon(t)\\
&\varepsilon(t) = y(t) - \boldsymbol{\psi}^T(t)\hat{\boldsymbol{\alpha}}(t-1)\\
&\boldsymbol{\psi}(t) = \boldsymbol{\varphi}(t) \otimes \mathbf{f}(t)\\
&\mathbf{f}(t) = \mathbf{A}\mathbf{f}(t-1)\\
&\boldsymbol{K}(t) = \frac{\boldsymbol{P}(t-1)\boldsymbol{\psi}(t)}{\lambda + \boldsymbol{\psi}^H(t)\boldsymbol{P}(t-1)\boldsymbol{\psi}(t)}\\
&\boldsymbol{P}(t) = \lambda^{-1}\left[\boldsymbol{P}(t-1) - \boldsymbol{K}(t)\boldsymbol{\psi}^H(t)\boldsymbol{P}(t-1)\right]
\end{aligned} \tag{9}$$

where $\mathbf{A} = diag\left\{e^{j\omega_1}, \ldots, e^{j\omega_k}\right\}$, and $\boldsymbol{P}(t) = \mathbf{R}^{-1}(t)$. In the WLS recursive algorithms, to avoid inversion of the correlation matrix $\mathbf{R}(t)$ at the initial phase of the estimation, we choose the following initial condition:

$$\hat{\boldsymbol{\alpha}}(0) = 0, \boldsymbol{P}(0) = \eta \mathbf{I}_{nk}$$

where η is a large positive constant number, which is a standard initialization for all EWLS-type algorithms [1].

2.1. Gradient-Basis Function (GBF) Estimator

It is well known that EWLS type algorithms such as EWBF, converge fast, but they are very demanding in terms of computational complexity. However, in contrast, the stochastic gradient algorithm is simple while both of them have similar parameter tracking capabilities. Therefore, using the Gradient method, we can estimate the parameter vector $\hat{\boldsymbol{\alpha}}(t)$, by minimizing the following error term:

$$\hat{\boldsymbol{\alpha}}(t) = \arg\min_{\boldsymbol{\alpha}} E\{\left| y(t) - \boldsymbol{\psi}^T(t)\boldsymbol{\alpha} \right|^2\}. \tag{10}$$

The GBF algorithm (11) can be obtained by replacing the inverse correlation matrix $\boldsymbol{P}$ in EWBF estimator, which requires (*nk*-by-*nk*) matrix updating at each iteration, with the scalar step size $\mu > 0$ [5]:

$$\begin{aligned}
&\hat{\boldsymbol{\theta}} = \mathbf{D}(t)\hat{\boldsymbol{\alpha}}(t) \\
&\hat{\boldsymbol{\alpha}}(t) = \hat{\boldsymbol{\alpha}}(t-1) + \mu \boldsymbol{\psi}^*(t)\varepsilon(t) \\
&\boldsymbol{\psi}(t) = \boldsymbol{\varphi}(t) \otimes \mathbf{f}(t) \\
&\mathbf{f}(t) = \mathbf{A}\mathbf{f}(t-1) \\
&\varepsilon(t) = y(t) - \boldsymbol{\psi}^T(t)\hat{\boldsymbol{\alpha}}(t-1).
\end{aligned} \tag{11}$$

3. RECURSIVE INVERSE BF (RIBF) ESTIMATOR

The newly proposed Recursive Inverse (RI) algorithm has been applied successfully in many signal processing areas such as image processing [6], ALE [7], and channel equalization. In this section, we propose the new Recursive Inverse Basis Function (RIBF) estimator and its adaptive-frequency version based on RI and the Gradient estimator approach.

3.1. RIBF-Estimator

Solving the Wiener-Hopf equation [1] leads to an optimum solution for the system coefficients given by

$$\hat{\boldsymbol{\alpha}}(t) = \mathbf{R}^{-1}(t)\mathbf{s}(t). \tag{12}$$

As the filter coefficients vector is updated, the solution of (12) is required at each iteration. Furthermore, there is a condition for the autocorrelation matrix to be nonsingular at each iteration [1]. Further, the Wiener-Hopf equation can be solved iteratively at each time step *k* leading to [7]:

$$\hat{\boldsymbol{\alpha}}_{k+1}(t) = \left[\mathbf{I} - \mu\mathbf{R}(t)\right]\hat{\boldsymbol{\alpha}}_k(t) + \mu\varepsilon(t)\mathbf{s}(t). \tag{13}$$

where μ satisfy the convergence criteria [1]

$$\mu < \frac{2}{\Lambda_{\max}(\mathbf{R}(t))}. \tag{14}$$

Considering the expectation of the estimated version of the autocorrelation matrix in (8)

$$\bar{\mathbf{R}}(t) = \lambda\bar{\mathbf{R}}(t-1) + \mathbf{R}_{\psi\psi}. \tag{15}$$

is obtained where $\mathbf{R}_{uu} = E\left\{\boldsymbol{\psi}(t)\boldsymbol{\psi}^T(t)\right\}$, $\bar{\mathbf{R}}(t) = E\left\{\mathbf{R}(t)\right\}$ .Solving the difference equation (15) yields

$$\bar{\mathbf{R}}(t) = \frac{1-\lambda^t}{1-\lambda}\mathbf{R}_{\psi\psi}. \tag{16}$$

Considering the maximum value, when $t \to \infty$

$$\bar{\mathbf{R}}(\infty) = \frac{1}{1-\lambda}\mathbf{R}_{\psi\psi}. \tag{17}$$

Eq. (16) states that the eigenvalues of the estimated correlation matrix $\bar{\mathbf{R}}(t)$ increase exponentially as $(1-\lambda^t)$, with a maximum limit $(1-\lambda)^{-1}$ times that of the original correlation matrix. Since the step-size μ is restricted to satisfy (14) we get

$$\mu < \frac{2(1-\lambda)}{\Lambda_{\max}(\mathbf{R}_{\psi\psi})} = \mu_{\max} \tag{18}$$

The former equation shows that the step-size μ takes values which are much smaller than that required in (14). It would be more convenient to replace the step size with a variable one so that

$$\mu(t) < \frac{2}{\Lambda_{\max}(\mathbf{R}(t))} = \left(\frac{1}{1-\lambda^t}\right)\left(\frac{2(1-\lambda)}{\Lambda_{\max}(\mathbf{R}_{\psi\psi})}\right). \tag{19}$$

Equivalently,

$$\mu(t) < \frac{\mu_{\max}}{1-\lambda^t} = \frac{\mu_0}{1-\lambda^t} \tag{20}$$

where $\mu_0 < \mu_{\max}$. It's clear in the steepest descent update equation (13) that there is a high complexity cost. Replacing the step-size constant with the variable one leads to only one iteration at each time step to be efficient [7]. Hence, the final RIBF algorithm becomes

$$\begin{aligned}
&\hat{\boldsymbol{\theta}} = \mathbf{D}(t)\hat{\boldsymbol{\alpha}}(t) \\
&\mu(t) = \frac{\mu_0}{1-\lambda^t} \\
&\hat{\boldsymbol{\alpha}}(t) = \left[\mathbf{I}_{nk} - \mu(t)\mathbf{R}(t)\right]\hat{\boldsymbol{\alpha}}(t-1) + \mu(t)\mathbf{s}(t) \\
&\boldsymbol{\psi}(t) = \boldsymbol{\varphi}(t) \otimes \mathbf{f}(t) \\
&\mathbf{f}(t) = \mathbf{A}\mathbf{f}(t-1) \\
&\mathbf{R}(t) = \lambda\mathbf{R}(t-1) + \boldsymbol{\psi}(t)\boldsymbol{\psi}^H(t) \\
&\mathbf{s}(t) = \lambda\mathbf{s}(t-1) + y(t)\boldsymbol{\psi}^*(t)
\end{aligned} \tag{21}$$

3.2. Frequency-Adaptive RIBF-Estimator

The proposed equalization algorithms would not be complete without a method for estimating and tracking the slight changes of the Doppler frequencies. Although the BF estimators are robust to small local changes in frequencies around known specified values, they fail to identify the system properly in the presence of a frequency drift [2].

Therefore, we adopt a simple gradient search strategy, which is proposed in [5], to derive the frequency-adaptive version of the RIBF algorithm. Let the gradient be

$$J(t,\boldsymbol{\omega}) = \frac{1}{2}\left|\varepsilon(t,\boldsymbol{\omega})\right|^2, \tag{22}$$

where $\boldsymbol{\omega} = [\omega_1, \ldots, \omega_k]^T$ are the instantaneous frequencies to be tracked. A simple gradient algorithm which minimizes (22) can be stated as

$$\hat{\boldsymbol{\omega}}(t+1) = \hat{\boldsymbol{\omega}}(t) - \mu \nabla J\left(\hat{\boldsymbol{\omega}}(t)\right) \quad (23)$$

where $\nabla J(\hat{\boldsymbol{\omega}}(t))$ denotes the gradient of $J(t, \boldsymbol{\omega})$ with respect to $\boldsymbol{\omega}$, evaluated at $\hat{\boldsymbol{\omega}}(t)$, and $\mu > 0$ is a small adaptation constant. In the case of frequency estimation, a modification in the general regression vector and the estimated parameters is needed in the form of arranging them according to their respective different frequency components ($\omega_1, \ldots, \omega_k$).

Let $\boldsymbol{\alpha}_l = [a_{1l}, \ldots, a_{nl}]^T$ be the vector of system coefficients corresponding to a particular frequency ω_l. Similarly, let $\boldsymbol{\psi}_l(t) = \boldsymbol{\varphi}(t) e^{j\omega_l t}$ be the generalized regression vector that corresponds to the lth frequency component. Accordingly, the error will be given as

$$\varepsilon(t) = y(t) - \sum_{l=1}^{k} \psi_l^T(t) \hat{\boldsymbol{\alpha}}_l(t-1) = y(t) - \boldsymbol{\psi}^T(t) \hat{\boldsymbol{\alpha}}_l(t-1) \quad (24)$$

where

$$\hat{\boldsymbol{\psi}}(t) = \left[\hat{\boldsymbol{\psi}}_1^T(t), \ldots, \hat{\boldsymbol{\psi}}_l^T(t)\right]^T, \; \hat{\boldsymbol{\alpha}}_l(t-1) = \left[\hat{\boldsymbol{\alpha}}_1^T(t), \ldots, \hat{\boldsymbol{\alpha}}_l^T(t)\right]^T.$$

Therefore,

$$\nabla J_l\left(\hat{\boldsymbol{\omega}}(t)\right) = \frac{\partial J(t, \boldsymbol{\omega})}{\partial \omega_l(t)} |_{\omega_l}$$
$$= \mathrm{Re}\left\{ j\varepsilon(t) e^{-j\omega_l} \boldsymbol{\varphi}^H(t) \hat{\boldsymbol{\alpha}}_l^*(t-1) \right\}$$
$$= \mathrm{Im}\left\{ \varepsilon^*(t) \hat{\boldsymbol{\psi}}_l^T(t) \hat{\boldsymbol{\alpha}}_l^*(t-1) \right\}$$

where $\hat{\boldsymbol{\psi}}_l(t) = \hat{f}_l(t) \boldsymbol{\varphi}(t), \quad \hat{f}_l(t) = e^{-j\hat{\omega}_l t} \hat{f}_l(t-1)$

The number and the values of system frequencies have to be initialized properly in order to avoid divergence of the frequency adaptive BF-estimators. In channel identification, the known (training) part can be employed for this purpose. The angular frequencies of the periodically-varying system can be initialized based on the analysis of higher order input/output signal statistics (these are further described in [3], [5]) or using the method of sliding window least squares estimates of system coefficients [8].

4. COMPUTAIONAL COMPLEXITY

Table (1) illustrates a comparison of the computational complexity for all the estimators that have been described. The computational complexity stands for the number of complex multiply/add operations per sampling time. In the evaluation, we considered the fact that some of the matrices involved are Hermitian (i.e. $\boldsymbol{P}$ and $\mathbf{R}$), hence, only their upper (lower) triangular parts have to be updated.

The proposed RIBF-algorithms have a clear computational advantage over the EWBF-estimator. It reduces the complexity by $kn(kn-1)$ multiply/add operations. Furthermore, it also has the capability to converge to smaller error values in terms of the mean square error (MSE), shown in the simulation section. Interestingly, despite this significant reduction in the proposed RIBF estimator, more reduction may be achieved, if we approximate the autocorrelation matrix as a Toeplitz matrix.

Table 1: Comparison of Computational Complexity of Different BF-Algorithms

Algorithm	Complexity
EWBF	$2(kn)^2 + 6kn + k$
GBF	$4kn + k + 1$
RIBF	$(kn)^2 + 7kn + k$

The Gradient estimator has the least computational complexity over all the others as is clear from Table 1. However, the price paid for reduced complexity of gradient algorithms is their initial slow convergence.

5. SIMULATION RESULTS

The test case, adopted from [2] and [5], involve a periodically time-varying channel as in (1) and (2). There are two channel taps (coefficients, n=2). Usually, the channel coefficient's number is a small number that depends on the transmission-channel's memory. Each channel-tap is equal to the linear combination of three basis functions (number of dispersive paths, k=3), given by:

$$f_1(t) = 1, f_2(t) = e^{j(2\pi/T_2)t}, f_3(t) = e^{j(2\pi/T_3)t}$$

where $T_2 = 120$ and $T_3 = 200$ sample periods. These numbers are chosen to be close to the real values meant for the carrier frequency $f_c = 900$ MHz, a bit rate around 20 kbit/s, and a vehicle moving at 100 km/h. The input signal is assumed to be 4-QAM (generated as $u(t) = \pm 1 \pm j$ with variance $\sigma_u^2 = 2$). The received signal $\hat{\boldsymbol{\theta}}$ is corrupted with AWGN ($\sigma_v^2 = 0.4373$). Consequently, the received signal has an average SNR equal to 15 dB. Fig.1 shows the development of the mean square parameter estimation error (ensemble average over 200 realizations) for the EWBF algorithm in (9), the GBF algorithm in (11), and the proposed RIBF estimator summarized in (21). All of the results in Fig.1 are for a fixed forgetting factor ($\lambda = 0.99$) corresponding to an estimation memory equal to approximately (58) samples. Also, the step size (adaptation constant) for the gradient algorithm was set to $\mu = 0.00572$; such a value has misadjustment equivalent in the steady state to the other BF's misadjustments), i.e., all the filters approximately have the same memory. Fig.1 demonstrates clearly the advantages of the RIBF estimator in terms of accuracy (by reaching MSE values for different step sizes (μ_0). The RIBF estimator outperforms EWBF by about 8 dB (at $\mu_0 = 0.00002$) and GBF by 12 dB. Even though, the estimators converge to the steady state MSE value at the same time $t = 200$, RIBF has a further reduction of the MSE about 1.3 dB over EWBF and 1.55 dB over the

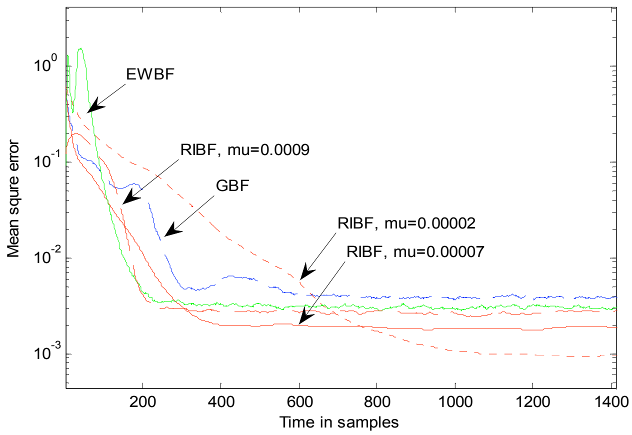

Fig.1: Comparison of Mean Square Parameter Estimation Error for Three Estimation Algorithms; Gradient-BF, EWBF and RIBF.

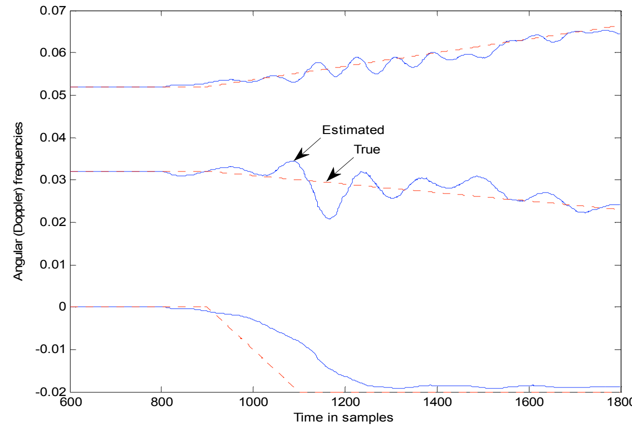

Fig. 2: Adaptive Frequency-RIBF Estimator Response, True Frequencies (dotted lines) and Their Estimates (solid lines).

Gradient-BF estimators. With the significant reduction in MSE, combined with low computational complexity, RIBF promises low BER without using any error correction code.

Fig. 2 illustrates the tracking frequency capabilities of the developed RIBF estimator. The simulation settings were exactly the same as in the previous example, except that the adaptation constant of the gradient search algorithm is set to be ($\mu = 0.00035$) and the other parameters of RIBF are set as($\lambda = 0.99$, $\mu_0 = 23\times10^{-6}$). The frequency drift starts at *t*=800 after the estimator has reached its steady-state.

6. CONCLUSIONS

The problem of identification and tracking of periodically varying systems has been considered. The classical basis function algorithms which were used to estimate and track time-varying coefficients in such systems have good tracking performance, yet they are computationally demanding. We have proposed a new recursive algorithm RIBF which outperforms the classical basis function schemes in terms of complexity reduction and tracking performance. It is superior to EWBF by reducing the complexity by $kn(kn-1)$ multiply/add operations and it shows further reduction in the mean square parameter estimation error by 8 dB. Furthermore, an adaptive-frequency version of the proposed algorithm was derived by employing a simple gradient search strategy.

APPENDIX A

The Kronecker product $\boldsymbol{X}\otimes\boldsymbol{Y}[im\times jn]$ of two matrices $\boldsymbol{X}[i\times j]$ and $\boldsymbol{Y}[m\times n]$ is defined as

$$\boldsymbol{X}\otimes\boldsymbol{Y} \begin{bmatrix} x_{11}\boldsymbol{Y} & \cdots & x_{1j}\boldsymbol{Y} \\ \vdots & \ddots & \vdots \\ x_{i1}\boldsymbol{Y} & \cdots & x_{ij}\boldsymbol{Y} \end{bmatrix}$$

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