

PERFORMANCE OF THE FREQUENCY-RESPONSE-SHAPED LMS ALGORITHM IN IMPULSIVE NOISE

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ABSTRACT

The performance of the Frequency-Response-Shaped Least Mean Square (FRS-LMS) adaptive algorithm in estimating a sinusoidal signal in impulsive and correlated noise is investigated. The algorithm does not require a priori knowledge about the nominal Gaussian process and is able to adapt to changes in the environment. The performance of the FRS-LMS is compared to that of the Leaky-LMS algorithms in terms of Mean Square Error (MSE) and convergence speed. The results indicate that while the FRS-LMS and the Leaky LMS algorithms perform similarly in AWGN, the FRS-LMS provides superior performance in impulsive and correlated noise environments. The performance gain is due to the frequency shaping and outlier reduction properties of the algorithm.

Index Terms— FRS-LMS, Leaky-LMS, correlated noise, impulsive noise.

1. INTRODUCTION

In many signal processing applications, suppressing noise, in order to estimate a known signal precisely, is of fundamental importance. The noise is usually modeled by using the Gaussian distribution due to its simplicity. However, physical observations indicate that the noise distribution shows Non-Gaussian behavior, such as man-made noise, underwater acoustic noise and atmospheric noise [1] [2]. This type of noise which has a heavy-tailed distribution is characterized by outliers and may be modeled using a Gaussian mixture model [3]. The pdf of the noise model can be described as [3][4]:

$$f = (1 - \epsilon)G(0, \sigma_n^2) + \epsilon G(0, \kappa \sigma_n^2), \quad (1)$$

where $G(0, \sigma_n^2)$ is a Gaussian pdf with zero mean and variance σ_n^2 and represents the nominal background noise. $G(0, \kappa \sigma_n^2)$ represents the impulsive component of the noise model where ϵ is the probability and $\kappa \geq 1$ is the strength of impulsive components. The general signal in noise model can be described as:

$$x(k) = s(k) + n(k), \quad (2)$$

where $s(k)$ is the desired signal and $n(k)$ is the impulsive noise component which has algebraic tails that are significantly heavier than the exponential tail of the Gaussian

distribution. Estimating $s(k)$ from $x(k)$ is an important problem. Adaptive filtering is one of the available methods, but has some convergence and complexity problems [5][6]. Wiener and Kalman filtering techniques require high implementation complexity [5]. Leaky-LMS algorithm [7], is simple and has good performance in impulsive noise environments even at relatively small signal-to-noise ratios (SNR's). The FRS-LMS algorithm has been proposed as a generalized version of the Leaky LMS in [8]. The algorithm has been shown to have fast convergence even in correlated Gaussian noise environments. In this paper, we investigate the performance of FRS-LMS algorithm in suppressing white and correlated impulsive noise and compare its performance with that of the Leaky-LMS algorithm.

2. SYSTEM MODEL AND DERIVATION

Consider the output of the adaptive transversal filter as:

$$y(k) = \mathbf{h}^H(k)\mathbf{x}(k), \quad (3)$$

where $\mathbf{h}(k)$ is the adaptive weight vector,

$$\mathbf{h}(k) = [h_0(k) \quad h_1(k) \quad \dots \quad h_{N-1}(k)]^T, \quad (4)$$

N is the filter length, and $\mathbf{x}(k)$ is the discrete time input signal given by

$$\mathbf{x}(k) = [x(k) \quad x(k-1) \quad \dots \quad x(N-k+1)]^T. \quad (5)$$

The update equation of the weight vector in the standard LMS algorithm is given as [5]:

$$\mathbf{h}(k+1) = \mathbf{h}(k) + \mu e^*(k)\mathbf{x}(k), \quad (6)$$

where $e(k)$ denotes the error, $*$ denotes complex conjugation and μ is the step size. $e(k)$ is given by:

$$e(k) = d(k) - y(k), \quad (7)$$

where $d(k)$ is the desired response of the filter, $y(k)$ is the output of the filter. In order to minimize the error, we start by establishing the cost function as:

$$J(k) = e^2(k), \quad (8)$$

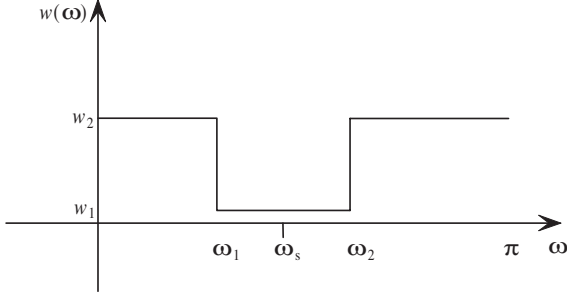


Figure 1. Weight Function.

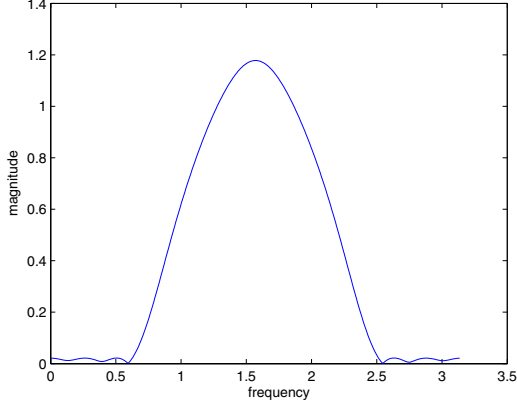


Figure 2. Frequency response of the noise filter.

The cost function in Leaky-LMS [5] is modified by adding a filter norm term to equation (8)

$$J_{leaky}(k) = e^2(k) + \gamma \mathbf{h}^H(k) \mathbf{h}(k), \quad (9)$$

where γ stands for the leakage factor.

2.1. The FRS-LMS Algorithm

In FRS-LMS, the leakage factor γ becomes a matrix [8]. Consider the total weighted noise power of the filter output at time step k :

$$G(k) = \int_0^\pi w_0(\omega) |H_k(\omega)|^2 d\omega, \quad (10)$$

where

$$H_k(\omega) = \sum_{n=0}^{N-1} h_n(k) e^{-j\omega n}, \quad (11)$$

is the frequency response of the filter at time k , $w_0(\omega)$ is a properly chosen weight function. If $w_0(\omega)$ becomes equal to the noise power spectral density (psd), then (10) becomes the total correlated noise power at the output of the filter. It is also assumed that $G(k)$ is constrained by an upper limit, P_0 . Combining (10), (11) and by using the method of Lagrange multipliers, the cost function to be minimized is obtained as:

$$J_{FRS}(k) = e^2(k) + \zeta(G(k) - P_0), \quad (12)$$

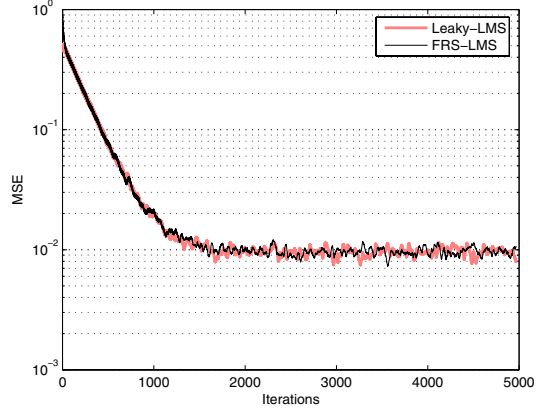


Figure 3. The ensemble MSE for FRS-LMS and Leaky-LMS in AWGN. $N = 32$, SNR= 5dB, $\mu = 0.00025$. Leaky-LMS: $\gamma = 0.001$, FRS-LMS: $\omega_b = \pi/3$, $w_1 = 20$, $w_2 = 0.002$.

and

$$\mathbf{h}(k+1) = \mathbf{h}(k) - \frac{1}{2} \mu \nabla J_{FRS}(k), \quad (13)$$

where $\nabla J_{FRS}(k)$ is the gradient of the cost function with respect to the vector $\mathbf{h}(k)$ and is given by:

$$\nabla J_{FRS}(k) = -2e^*(k) \mathbf{x}(k) + 2\zeta \mathbf{F}_0 \mathbf{h}(k), \quad (14)$$

where $G(k)$ is a matrix given by:

$$G(k) = \mathbf{h}^H(k) \mathbf{F}_0 \mathbf{h}(k), \quad (15)$$

In (15), \mathbf{F}_0 is a matrix with elements determined as:

$$f_0(n, m) = \int_0^\pi w_0(\omega) \cos[(n-m)\omega] d\omega, \quad (16)$$

Finally using (13) and (14), the filter coefficient vector update equation becomes:

$$\mathbf{h}(k+1) = [\mathbf{I} - \mu \mathbf{F}] \mathbf{h}(k) + \mu e^*(k) \mathbf{x}(k), \quad (17)$$

where $\mathbf{F} = \zeta \mathbf{F}_0$. For convergence, the step size μ should be constrained as,

$$\mu < \frac{2}{\lambda_{\max}(\mathbf{R})} \quad (18)$$

where λ_{\max} is the maximum eigenvalue of \mathbf{R} . $\mathbf{R} = \mathbf{F} + \mathbf{R}_{xx}$, where \mathbf{R}_{xx} is the autocorrelation matrix of the input signal. It has also been shown in [8] that the error variance of the FRS-LMS algorithm can be made less than that of the standard LMS, provided that $\mu < \frac{1}{\lambda_{\max}}$. Using the fast computation method in [8], the overall complexity of the proposed algorithm is comparable to that of the Leaky-LMS algorithm.

3. SIMULATION RESULTS

In the simulations, it is assumed that a sinusoidal signal $s(k)$ with frequency $\omega_s = \pi/3$ is corrupted by additive noise. The cases where the noise contains impulsive components and when it becomes correlated are investigated in detail.

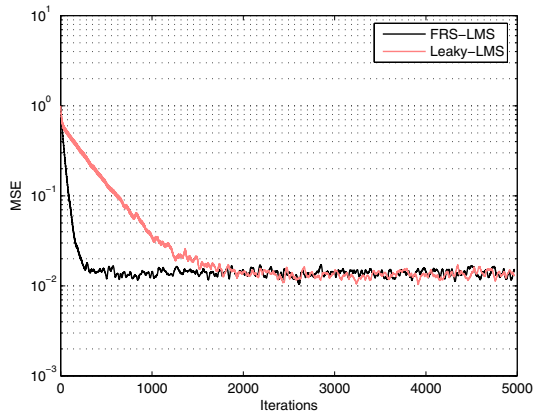


Figure 4. The ensemble MSE for FRS-LMS and Leaky-LMS in ACGN, $N = 32$, SNR= 5dB. Leaky-LMS: $\mu = 0.00016$, $\gamma = 0.0001$. FRS-LMS: $\mu = 0.0015$, $\omega_b = \pi/6$, $w_1 = 10$, $w_2 = 0.001$.

3.1. Input signal with Additive Gaussian Noise

3.1.1. Additive White Gaussian Noise (AWGN)

The signal $s(k)$ is assumed to be corrupted by AWGN. Simulations of the FRS-LMS algorithm were done using the fast computation method in [8] with $\mu = 0.00025$, filter length $N = 32$ taps. The weight function used in the simulations is shown in Fig. 1 with window size ($\omega_b = \omega_2 - \omega_1$), $\omega_b = \pi/3$, $w_1 = 20$, $w_2 = 0.002$ and SNR= 5dB. For the Leaky-LMS algorithm, the parameters used were; $\mu = 0.00025$, $\gamma = 0.001$. Fig. 3 shows that Leaky-LMS and the FRS-LMS algorithms have similar speed of convergence (they both converge after 1600 time samples) for the same MSE (MSE=0.01). This is expected as the FRS-LMS can be considered as the generalized version of the Leaky-LMS and the frequency-response-shaping capability does not provide any advantage in AWGN.

3.1.2. Additive Correlated Gaussian Noise (ACGN)

A correlated Gaussian noise process is generated and added to the received signal $s(k)$. The correlated noise was generated by filtering an AWGN process with an FIR filter having the magnitude response characteristic shown in Fig. 2. Simulation parameters for FRS-LMS are similar with $\mu = 0.0015$, $\omega_b = \pi/6$ and $w_1 = 10$, $w_2 = 0.001$, SNR= 5dB. The Leaky-LMS algorithm has $\mu = 0.00016$, $\gamma = 0.0001$. Fig. 4 shows that for the same MSE (MSE=0.014), the FRS-LMS converges after about 300 time samples, whereas the Leaky-LMS converges after 1800 time samples. Here, the FRS-LMS is able to shape the frequency response in order to suppress correlated noise more effectively.

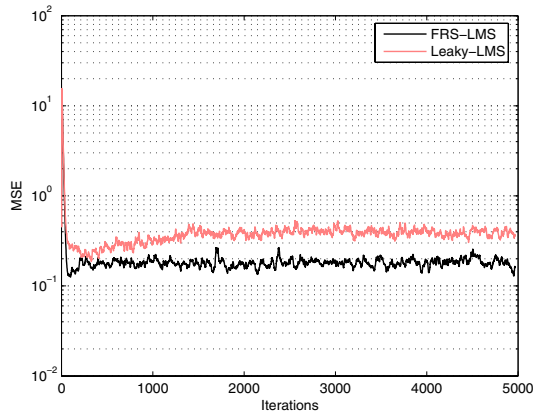


Figure 5. The ensemble MSE for FRS-LMS and Leaky-LMS in white impulsive noise, $\epsilon = 0.2$, $\kappa = 100$, $N = 32$, SNR= 5dB, $\mu = 0.00025$. Leaky-LMS: $\gamma = 0.001$. FRS-LMS: $\omega_b = \pi/6$, $w_1 = 20$, $w_2 = 0.001$.

3.2. Input Signal with Impulsive Noise

3.2.1. White Impulsive Noise

In order to study the effects of the impulsive components (outliers) of the noise process, a white impulsive noise process is generated with $\epsilon = 0.2$, $\kappa = 100$. These parameters are used to model severely impulsive noise [3]. The FRS-LMS algorithm has the parameters $\mu = 0.00025$, $\omega_b = \pi/6$ and $w_1 = 20$, $w_2 = 0.001$ and SNR= 5dB whereas for the Leaky-LMS algorithm, $\mu = 0.00025$, $\gamma = 0.001$. Fig. 5 shows that the FRS-LMS converges to a relatively lower MSE (MSE=0.19) than the Leaky-LMS (MSE=0.42). By comparing Fig. 3 and Fig. 5, it is also important to note that the outliers in the noise process has decreased the performance of both algorithms severely (approximately 20 and 40 times more MSE for FRS-LMS and Leaky LMS respectively). However, the FRS-LMS is relatively more robust to impulsive noise since the magnitudes of the outliers are reduced by the weight function.

3.2.2. Correlated Impulsive Noise

In order to study the effects of correlated impulsive noise on convergence, a correlated impulsive noise process is generated with $\epsilon = 0.2$, $\kappa = 100$. The FRS-LMS algorithm has the parameters $\mu = 0.00025$, $\omega_b = \pi/6$ and $w_1 = 100$, $w_2 = 0.001$, SNR= 5dB and the Leaky-LMS algorithm has $\mu = 0.00025$, $\gamma = 0.001$. Fig. 6 shows that for the same convergence speed, the FRS-LMS converges to a lower MSE (MSE=0.24) than the Leaky-LMS (MSE=0.62). The FRS-LMS is able to reduce the effects of the outliers and the correlated noise by shaping the frequency response. When we compare Fig. 5 and Fig. 6, it can be observed that due to the correlated noise (in the presence of outliers) the MSE increases approximately 1.25 and 1.5 times for FRS-LMS and Leaky LMS respectively. In the Gaussian mixture noise model for impulsive noise, the SNR is assumed to be constant, and in-

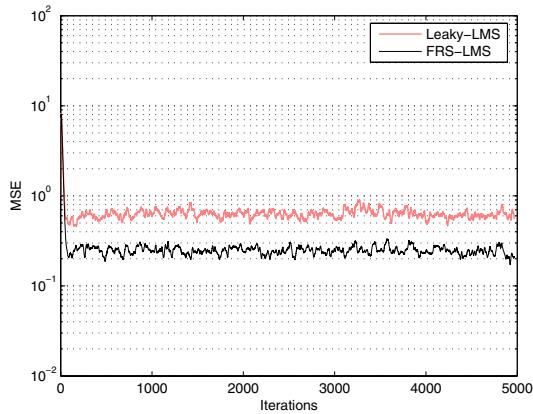


Figure 6. The ensemble MSE for FRS-LMS and Leaky-LMS in correlated impulsive noise, $\epsilon = 0.2$, $\kappa = 100$, $N = 32$, SNR= 5dB, $\mu = 0.00025$. Leaky-LMS: $\gamma = 0.001$. FRS-LMS: $\omega_b = \pi/6$, $w_1 = 20$, $w_2 = 0.001$.

creasing the impulsive component strength κ or the outlier frequency ϵ leads to an increase in the total noise power. An alternative approach [3], is to keep the total noise variance constant in order to study the effects of the variation in the shape of the noise distribution. In other words, when κ is increased for fixed ϵ , the nominal noise variance has to be decreased and vice versa. Figure 7 shows the performance of the FRS-LMS and the Leaky-LMS under the fixed noise variance assumption. The FRS-LMS algorithm has the parameters $\mu = 0.0015$, $\omega_b = \pi/6$ and $w_1 = 10$, $w_2 = 0.001$, SNR= 5dB whereas the Leaky-LMS algorithm has, $\mu = 0.00016$ and $\gamma = 0.0001$. When the MSE is fixed (MSE=0.015), the FRS-LMS converges much faster than the Leaky-LMS, (FRS-LMS converges after 300 time samples, whereas the Leaky-LMS converges after 1900 time samples). When Fig. 4 and Fig. 7 are compared, it is noticed that in the same correlated noise, the effect of constrained impulsive noise is very small. This is due to the fact that in constrained impulsive noise case, increasing the probability (ϵ) of the impulsive components will lead to a corresponding decrease in the background noise.

4. CONCLUSIONS

The performance of the Frequency-Response-Shaped Least Mean Square (FRS-LMS) adaptive algorithm in estimating a sinusoidal signal in impulsive and correlated noise is investigated. The FRS-LMS can be considered as a generalized version of the Leaky-LMS algorithm and has a similar computational complexity. By shaping the frequency response and reducing the effects of outliers the FRS-LMS algorithm shows robust performance in impulsive and correlated noise. The results indicate that while the FRS-LMS and the Leaky LMS algorithms perform similarly in AWGN, the FRS-LMS provides superior performance in impulsive and correlated noise environments.

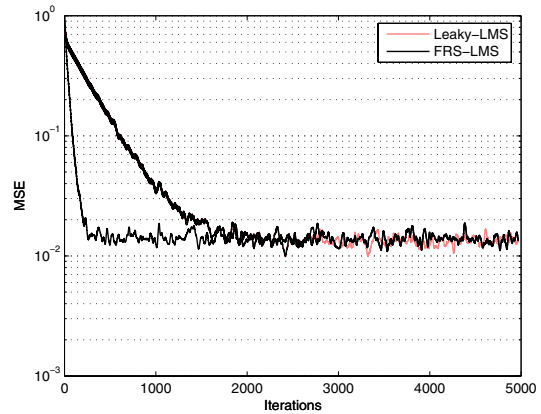


Figure 7. The ensemble MSE for FRS-LMS and Leaky-LMS in constrained correlated impulsive noise, $\epsilon = 0.2$, $\kappa = 100$, $N = 32$, SNR= 5dB. Leaky-LMS: $\mu = 0.00016$, $\gamma = 0.0001$. FRS-LMS: $\mu = 0.0015$, $\omega_b = \pi/6$, $w_1 = 10$, $w_2 = 0.001$.

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