Recursive Inverse Adaptive Filter with Second Order Estimation of Autocorrelation Matrix

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***Abstract*—The recently proposed Recursive Inverse (RI) Adaptive Filtering algorithm uses a variable step-size and the first order recursive estimation of the correlation matrices in the coefficient update equation which lead to an improved performance. In this paper, a new FIR adaptive filtering algorithm is introduced. This algorithm uses the second order recursive estimation of the correlation matrices in the coefficient update equation which leads to an improved performance over the RI algorithm. The simulation results show that the algorithm outperforms the Transform Domain LMS with Variable Step-Size (TDVSS), the RI and the RLS algorithms in stationary environments. The performance of the algorithms is tested in Additive White Gaussian Noise (AWGN) and Correlated Noise environments.*

***Keywords*—RI, RLS, TDVSS.*

I. Introduction

Adaptive filtering techniques are widely used to cope with the variations of the system parameters [1]. Maximum likelihood estimation techniques depend on the validity of some statistical nature of the signals. In many applications, due to the lack of statistical information these methods are not applied directly. The min-max estimation methods make no assumptions on the statistical nature of the signals. One example of this class of algorithms is the Least-Mean-Square (LMS) algorithm [2].

The LMS adaptive algorithm has been used in many application areas, such as channel equalization [3], system identification [4], adaptive array processing, and adaptive noise cancellation [1]. The FIR filter weights are updated iteratively by the minimization of the mean square error (mse) of the difference between filter output and the desired response.

The LMS algorithm is not only simple in its filter weight updating and hardware implementation but also reasonably fast in convergence if optimal step-size is used [5]. A number of LMS variations have been proposed to overcome the disadvantages of a fixed step-size parameter by employing variable step-size or automatic gain control schemes. The VSSLMS algorithm [6] is one of these algorithms. However, the LMS algorithms and its variants fail or perform poorly in some environments, especially, when the input signal is highly correlated or the additive noise is impulsive. The transform domain adaptive filtering algorithms such as the TDVSS algorithm [7] are effective in correlated noise environments.

The Recursive-Least-Squares (RLS) algorithm [8] was proposed to offer superior performance compared to that of the LMS algorithm and its variants, especially in highly correlated environments.

Although the RLS algorithm provides good performance in such environment, it suffers from its relatively high computational complexity. Also the RLS algorithm and its variants may face numerical stability problems due to the update of the inverse autocorrelation matrix [9]. In addition to that, in the RLS algorithm, the forgetting factor, β, has to be chosen such that its value is very close to unity to ensure the stability and convergence of the algorithm. However, this poses a limitation for the use of the algorithm since small values of β may be required for signal tracking in nonstationary environments. par The recently proposed recursive Inverse (RI) algorithm [10], which uses a variable step-size and the instantaneous value of the autocorrelation matrix in the coefficient update equation, leads to an improved performance with reduced complexity.

Even though the RI algorithm has better performance than the RLS algorithm, this performance can be improved by considering the second order update of the correlations with slight increment in the number of Add./Sub.

II. Second Order Recursive Inverse Adaptive Algorithm

In the recently proposed Recursive Inverse (RI) adaptive filtering algorithm [10], the correlations are estimated recursively [11],

$$\mathbf{R}(k) = \beta\mathbf{R}(k-1) + \mathbf{x}(k)\mathbf{x}^T(k), \tag{1}$$

$$\mathbf{p}(k) = \beta\mathbf{p}(k-1) + d(k)\mathbf{x}(k), \tag{2}$$

where β is the forgetting factor which is usually very close to one.

the update equation of the filter tap vector in the RI algorithm is given as [10],

$$\mathbf{w}(k) = [\mathbf{I} - \mu(k)\mathbf{R}(k)]\mathbf{w}(k-1) + \mu(k)\mathbf{p}(k). \quad (3)$$

If $\mu(k)$ in (3) satisfies the convergence criterion [5] it will be given by:

$$\begin{aligned} \mu(k) < \frac{2}{\lambda_{max}(\mathbf{R}(k))} &= \left(\frac{1-\beta}{1-\beta^k}\right)\left(\frac{2}{\lambda_{max}(\mathbf{R}_{xx})}\right) \\ &= \frac{\mu_{max}}{1-\beta^k}, \end{aligned} \quad (4)$$

where $\mathbf{R}_{xx} = E\left\{\mathbf{x}(k)\mathbf{x}^T(k)\right\}$ and $\bar{\mathbf{R}}(k) = E\{\mathbf{R}(k)\}$. Or equivalently,

$$\mu(k) = \frac{\mu_0}{1-\beta^k} \qquad \text{where } \mu_0 < \mu_{max}. \quad (5)$$

The RI algorithm has a major advantage compared to the RLS algorithm in that it does not require the updating of the inverse autocorrelation matrix. Also, its computational complexity is less than that of the RLS algorithm. Due to the update of inverse autocorrelation matrix, RLS type algorithms may face numerical stability problems [11], which is not the case for the RI algorithm. The computational complexity of the RI algorithm can be reduced further by implementing the fast implementation technique described in [10].

The Performance of the RI algorithm may be improved further by using the second order recursive updating of the correlations as

$$\mathbf{R}(k) = \beta_1\mathbf{R}(k-1) + \beta_2\mathbf{R}(k-2) + \mathbf{x}(k)\mathbf{x}^T(k), \quad (6)$$

$$\mathbf{p}(k) = \beta_1\mathbf{p}(k-1) + \beta_2\mathbf{p}(k-2) + d(k)\mathbf{x}(k), \quad (7)$$

The number of multiplications in the second order equations will not be increased compared with the first order updating equations if the coefficients in (6) and (7) are chosen to be equal, i.e. $\beta_1 = \beta_2 = \frac{1}{2}\beta$.

Taking the expectation of (6) gives

$$\bar{\mathbf{R}}(k) = \frac{1}{2}\beta\bar{\mathbf{R}}(k-1) + \frac{1}{2}\beta\bar{\mathbf{R}}(k-2) + \mathbf{R}_{xx}, \quad (8)$$

where $\mathbf{R}_{xx} = E\left\{\mathbf{x}(k)\mathbf{x}^T(k)\right\}$ and $\bar{\mathbf{R}}(k) = E\{\mathbf{R}(k)\}$.
Poles of the system in (8) are:

$$\begin{aligned} z_1 &= \frac{1}{4}\left(\beta - \sqrt{\beta^2 + 8\beta}\right) \\ z_2 &= \frac{1}{4}\left(\beta + \sqrt{\beta^2 + 8\beta}\right) \end{aligned} \quad (9)$$

which have magnitudes less than unity if $\beta < 1$. Solving (8)with the initial conditions $\bar{\mathbf{R}}(-2) = \bar{\mathbf{R}}(-1) = \bar{\mathbf{R}}(0) = \mathbf{0}$ yields,

$$\bar{\mathbf{R}}(k) = \left(\frac{1}{\beta-1} + \alpha_1 z_1^k + \alpha_2 z_2^k\right)\mathbf{R}_{xx}, \quad (10)$$

where

$$\begin{aligned} \alpha_1 &= \frac{\beta - z_2}{(1-\beta)(z_2 - z_1)} \\ \alpha_2 &= \frac{\beta - z_1}{(1-\beta)(z_2 - z_1)}. \end{aligned} \quad (11)$$

Letting,

$$\gamma(k) = \frac{1}{\beta-1} + \alpha_1 z_1^k + \alpha_2 z_2^k, \quad (12)$$

then, in the RI algorithm, the variable step-size is chosen as:

$$\mu(k) = \frac{\mu_0}{\gamma(k)}. \quad (13)$$

Where the update equation of the tap weight vector will remain the same as in (3).

III. Simulation Results

In the simulations, we are comparing the performance of the proposed algorithm with that of the RI algorithm [10] RLS [8] and TDVSS [7] algorithms in the system identification problem described in [7] in terms of mse. All the algorithms were implemented using a filter length of $N = 16$ taps.

A. Additive White Gaussian Noise

To test the performance of the proposed algorithm, the signal is assumed to be corrupted with an additive white Gaussian noise (AWGN) process with zero mean and variance ($\sigma_v^2 = 0.000225$). Simulations were done with the following parameters: For the 2^{nd} order RI algorithm, $\beta = 0.997$, $\mu_0 = 0.15$. For the RI algorithm, $\beta = 0.991$, $\mu_0 = 0.00146$. For the RLS algorithm, $\beta = 0.991$. For the TDVSS algorithm, $\alpha = 0.99$, $\beta = 0.9$, $\epsilon = 0.025$, $\mu_{min} = 0.0047$, $\mu_{max} = 0.05$, $\gamma = 0.001$, $L = 10$. Fig.1 shows that even though the RLS algorithm is converging faster at first, the RI and the RLS algorithms converge finally to the same mean-square error (mse=-50dB) at approximately 400 iterations. Also the 2^{nd} order RI algorithm converges with them at the same time but it continues to converge to a much lower mse (mse=-59dB) which shows the advantage of the 2^{nd} order RI algorithm over the other algorithms. the TDVSS converges to a relatively higher mse compared to that of the other algorithms (mse=-40dB). On the other hand, it should be noted that the computational complexity of the proposed algorithm is much lower than that of the RLS algorithm and very comparable with that of the RI algorithm.

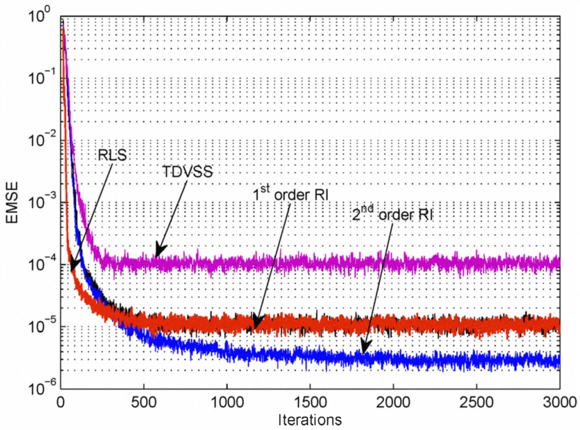

Fig. 1. The ensemble MSE for 2^{nd} Order RI, RI, RLS and TDVSS in AWGN, $\sigma_v^2 = 0.000225$, $N = 16$. 2^{nd} order RI: $\beta = 0.997$, $\mu_0 = 0.15$. RI: $\beta = 0.991$, $\mu_0 = 0.00146$. RLS: $\beta = 0.991$. TDVSS: $\alpha = 0.99$, $\beta = 0.9$, $\epsilon = 0.025$, $\mu_{min} = 0.0047$, $\mu_{max} = 0.05$, $\gamma = 0.001$, $L = 10$.

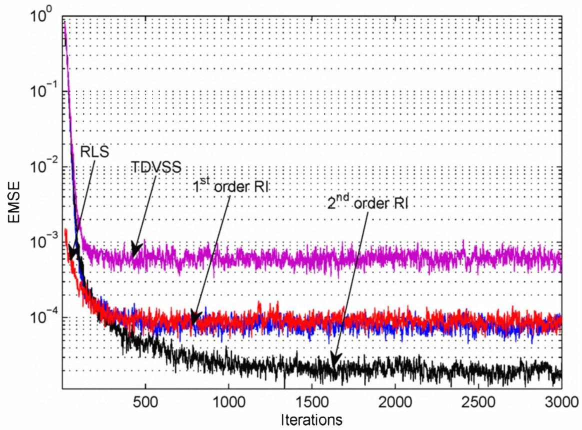

Fig. 2. The ensemble MSE for 2^{nd} Order RI, RI, RLS and TDVSS in ACGN, $\sigma_v^2 = 0.000576$, $N = 16$. 2^{nd} order RI: $\beta = 0.997$, $\mu_0 = 0.15$. RI: $\beta = 0.991$, $\mu_0 = 0.00146$. RLS: $\beta = 0.991$. TDVSS: $\alpha = 0.99$, $\beta = 0.9$, $\epsilon = 0.025$, $\mu_{min} = 0.0047$, $\mu_{max} = 0.05$, $\gamma = 0.001$, $L = 10$.

B. Additive Correlated Gaussian Noise

In order to test the robustness of the algorithms mentioned above due to tha changes of the environment, the input signal signal $x(n)$ is assumed to be corrupted by an additive correlated Gaussian noise (ACGN). A correlated Gaussian noise process is generated by using the first-order autoregressive model (AR(1)) $v(k+1) = \rho v(k) + v_0(k)$, where $v_0(k)$ is a white Gaussian noise process with zero mean and variance ($\sigma_v^2 = 0.000576$) and ρ is the correlation parameter ($\rho = 0.7$). Simulations were done with the following parameters: For the 2^{nd} order RI algorithm, $\beta = 0.997$, $\mu_0 = 0.15$. For the RI algorithm, $\beta = 0.991$, $\mu_0 = 0.00146$. For the RLS algorithm, $\beta = 0.991$. For the TDVSS algorithm, $\alpha = 0.99$, $\beta = 0.9$, $\epsilon = 0.025$, $\mu_{min} = 0.0047$, $\mu_{max} = 0.05$, $\gamma = 0.001$, $L = 10$. Fig.2 shows that the RLS algorithm is converging faster at the beginning, but the RI and the RLS algorithms converge finally to the same mean-square error (mse=-40dB) at approximately 400 iterations. Also the 2^{nd} order RI algorithm converges with them at the same time but it continues to converge to a much lower mse (mse=-49dB) which shows that the 2^{nd} order RI algorithm is slightly less affected by the noise type than the other algorithms. the TDVSS converges to a higher mse compared to that of the other algorithms (mse=-34dB).

IV. Conclusion

The proposed 2^{nd} order Recursive Inverse (RI) Adaptive Filtering algorithm uses a variable step-size and the 2^{nd} order recursive estimation of the correlation matrices in the coefficient update equation which lead to an improved performance. The simulation results show that the algorithm performs better than the TDVSS algorithm in terms of mse with approximately 18dB difference in AWGN and 15dB difference in ACGN environments, respectively. Also, with slight increment of of the RI computational complexity the proposed algorithm provides an advantage over the RI and RLS algorithms in terms of mse by 8dB in AWGN and 9dB in ACGN environments, respectively. On the other hand, the proposed algorithm does not require the update of the inverse autocorrelation matrix as the case of RLS algorithms which guarantees its stability.

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