A Weighted Zero-Attracting Leaky-LMS Algorithm

Mohammad Shukri Salman
Electrical and Electronic Engineering Dept.
Mevlana (Rumi) University, Selcuklu
Konya, Turkey
Email: mssalman@mevlana.edu.tr

Mohammad Naser Sabet Jahromi, Aykut Hocanin and Osman Kukrer
Electrical and Electronic Engineering Dept.
Eastern Mediterranean University, Famagusta
TRNC, Mersin 10, Turkey
{mohammad.sabet,aykut.hocanin,osman.kukrer}@emu.edu.tr

***Abstract*—In this paper, a novel weighted zero-attracting leaky-LMS (WZA-LLMS) adaptive algorithm for sparse systems is proposed. In the proposed algorithm, a log-sum penalty is incorporated into the cost function of the leaky-LMS algorithm, which results in a shrinkage in the update equation. This shrinkage gives the algorithm the ability of attracting zeros, i.e., when the system is sparse, and hence improves its performance. The performance of the proposed WZA-LLMS algorithm is compared to those of the standard leaky-LMS and ZA-LMS algorithms in sparse system identification settings. The WZA-LLMS algorithm shows superior performance compared to the algorithms.**

I. Introduction

The Least Mean Square (LMS) algorithm [1], is widely used in adaptive system identification. The performance of the LMS algorithm deteriorates when the input signal is highly correlated. Many LMS variants have been proposed to overcome this problem, i. e., leaky-LMS algorithm. In the leaky-LMS algorithm, a leakage factor is introduced to stabilize the system [2] and enhance its performance. It has been applied in many applications; such as system identification, adaptive noise cancellation [1], etc.

In many scenarios, the impulse response of the unknown system can be assumed to be sparse, containing only a few relatively large coefficients among many negligible ones. Using such sparse prior information can improve the filtering performance. However, the standard leaky-LMS filter does not exploit such information. Recently, many algorithms exploiting sparsity based on applying a subset selection scheme during the filtering process, which was implemented via statistical detection of active taps [3], [4] or sequential partial updating [5], were proposed.

Recently, a zero-attracting LMS (ZA-LMS) algorithm is proposed [6] for sparse systems. This algorithm employs a zero attraction mechanism to attract zero (or relatively small) system taps. The algorithm is shown to provide higher performance compared to the standard LMS algorithm. The main drawback of this algorithm is the poor performance when the input signal is correlated.

In this paper, we propose an alternative approach to identify sparse systems using leaky-LMS filters. The basic idea is to introduce a penalty, in the cost function of the leaky-LMS algorithm, which favors sparsity. This in turn results in a shrinkage in the update formula. This shrinkage enhances the performance of the adaptive filter when the majority of coefficients are zero; i.e. the system is sparse. Experimental results illustrate that the proposed filter exceeds the standard leaky-LMS in both transient and steady-state performance for sparse systems.

The paper is organized as follows. In section II, the proposed WZA-LLMS algorithm is derived. In section III, simulation results that compare the performance of the proposed algorithm with those of the standard leaky-LMS and ZA-LMS algorithms in sparse systems, are shown. Finally, in the last section, conclusions are drawn.

II. Zero-Attracting Leaky-LMS Algorithm

A. Leaky-LMS Algorithm

Consider a linear system with its input $x(n)$ and output $d(n)$ related by

$$d(n) = \mathbf{h}^T\mathbf{x}(n) + v(n) \tag{1}$$

where $\mathbf{h}$ is the impulse response of the unknown system with length L taps, $\mathbf{x}(n)$ is the system input tap vector and $v(n)$ is the additive noise which is independent from $x(n)$.

The leaky LMS algorithm minimizes the instantaneous objective function

$$J(n) = e^2(n) + \gamma\mathbf{w}^T(n)\mathbf{w}(n) \tag{2}$$

where $\mathbf{w}(n)$ is the coefficient weight vector of the adaptive algorithm with length L, γ is a positive parameter called the leakage factor and and $e(n)$ is the error signal given by

$$e(n) = d(n) - \mathbf{w}^T(n)\mathbf{x}(n) \tag{3}$$

The minimum of $J(n)$ can be sought recursively using the gradient method [7]

$$\begin{aligned}\mathbf{w}(n+1) &= \mathbf{w}(n) - \frac{\mu}{2}\frac{\partial J(n)}{\partial \mathbf{w}(n)} \\ &= (1-\mu\gamma)\mathbf{w}(n) + \mu e(n)\mathbf{x}(n)\end{aligned} \tag{4}$$

where μ is the adaptation step-size.

B. Proposed Algorithm

In the weighted zero-attracting leaky-LMS (WZA-LLMS) algorithm, a log-sum penalty of the coefficient vector is added to the the cost function of the original leaky-LMS algorithm [2]. So the new cost function is defined by

$$J1(n) = e^2(n) + \gamma \mathbf{w}^T(n)\mathbf{w}(n) + \gamma' \sum_{i=1}^{L} log\left(1 + \frac{|w_i|}{\zeta'}\right) \quad (5)$$

where γ' and ζ' are positive constants. Using the gradient method, the update equation of the WZA-LLMS algorithm becomes

$$w_i(n+1) = (1-\mu\gamma)w_i(n) + \mu e(n)x_i(n) - \rho\frac{sgn\left[w_i(n)\right]}{1+\zeta|w_i(n)|} \quad (6)$$

or equivalently, in vector form

$$\mathbf{w}(n+1) = (1-\mu\gamma)\mathbf{w}(n) + \mu e(n)\mathbf{x}(n) - \rho\frac{sgn\left[\mathbf{w}(n)\right]}{1+\zeta|\mathbf{w}(n)|} \quad (7)$$

where $\rho = \frac{\mu\gamma'}{\zeta'}$ is the zero-attracting parameter, $\zeta = \frac{1}{\zeta'}$ and $sgn(.)$ is a component-wise sign function defined by

$$sgn(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases} \quad (8)$$

Comparing (4) and (7), we note that the WZA-LLMS algorithm has an additional term $\left(-\rho\frac{sgn[\mathbf{w}(n)]}{1+\zeta|\mathbf{w}(n)|}\right)$ which always attracts the tap coefficients to zero. This is called a zero -attractor because its strength is controlled by ρ. In other words, it will speed-up convergence when most of the system coefficients are zero, i.e., the system is sparse.

III. Simulation Results

In this section, the performance of the WZA-LLMS is compared with those of the standard leaky-LMS and ZA-LMS algorithms in a sparse system identification setting. All the experiments are implemented with 100 independent Monte-Carlo runs. All the parameters used were selected by extensive simulations to give optimal performance for each algorithm.

In the first experiment, in order to exploit the sparsity of the system, we use a filter of 16 coefficients in the time varying system [8]. Initially, the one random tap of the unknown system is set to 1 and the others to zero; to have a sparsity of 1/16. After 500 iterations, all the odd taps are set to 1 and all the even taps are kept to be zero, i.e., a sparsity of 8/16. Finally, after 1000 iterations all the even taps are set with value -1 and all the odd taps are maintained to be 1, leaving a completely non-sparse system. The input signal and the observed noise are both assumed to be white Gaussian random sequences with variances 1 and 10^{-3}, respectively, in order to have a 30dB signal-to-noise ratio (SNR). The performance measure used is the mean-square deviation (MSD) defined as $MSD = E\|\mathbf{h} - \mathbf{w}(n)\|^2$. Simulations are done with the following parameters: For the leaky-LMS: $\mu = 0.035$, $\gamma = 0.001$. For the WZA-LLMS: $\mu = 0.035$, $\gamma = 0.001$, $\zeta = 10$ and $\rho = 5 \times 10^{-4}$. For ZA-LMS: $\mu = 0.035$ and $\rho = 5 \times 10^{-4}$. Fig. 1 shows the average MSD estimate of both algorithms. As seen from the MSD results, when the system

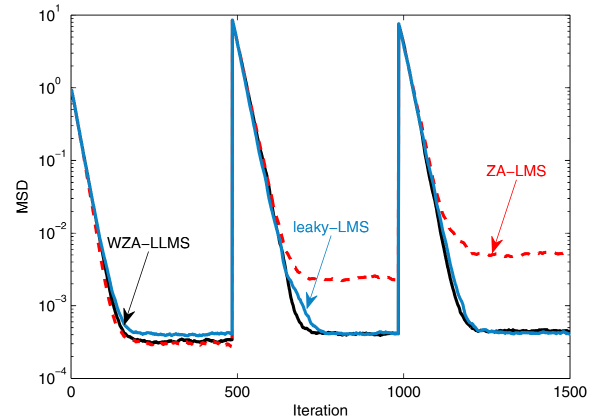

Fig. 1. Tracking and steady-state behaviors of a 16 tap adaptive filter driven by a white input signal.

is very sparse (before the 500^{th} iteration), the WZA-LLMS converges at the same rate of the other algorithms with better steady-state MSD than the leaky-LMS (1dB better). After the 500^{th} iteration, as the number of non-zero taps increases to eight, we see that the WZA-LLMS algorithms converges to the same MSD as that of the leaky-LMS but with 30 iterations faster rate. Also, it converges slightly faster than the ZA-LMS algorithm but with 9.5dB better MSD performance. After 1000 iterations, where the system is now completely non-sparse, the WZA-LLMS algorithm performs exactly like the leaky-LMS and much better than the ZA-LMS algorithm.

In the second experiment, the same system is used, except that the switching times are set to the 5000^{th} iteration and the 10000^{th} iteration, respectively. The input signal $x(n)$ is now a correlated signal generated by the AR(1) process ($x(n) = 0.8x(n-1) + v_o(n)$) and then normalized to variance 1 (in order to keep the same signal-to-noise ratio as the first experiment), where $v_o(n)$ is a white Gaussian process. The observed noise is the same noise assumed in the first experiment with the same parameters. Simulations are done with the following parameters: For the leaky-LMS: $\mu = 0.015$, $\gamma = 0.0001$. For the WZA-LLMS: $\mu = 0.015$, $\gamma = 0.0001$, $\zeta = 10$ and $\rho = 2 \times 10^{-4}$. For ZA-LMS: $\mu = 0.015$ and $\rho = 3 \times 10^{-5}$. Fig. 2 shows the average MSD estimate of all algorithms.ig. 2 shows the average MSD estimate of both algorithms. As seen from the MSD results, when the system is very sparse, the WZA-LLMS converges to the same MSD as that of ZA-LMS with the same rate and faster than the leaky-LMS (400 iterations faster) with 3.5dB better MSD. As the number of non-zero taps increases to eight, we see that the WZA-LLMS algorithms converges faster than both algorithms with 0.5dB better MSD. When the system is completely non-sparse, the WZA-LLMS algorithm still outperforms both algorithms.

The third experiment simulates an acoustic echo cancellation problem. The driving signal and the observed noise are the same as the first experiment. Fig. 3 shows an acoustic echo path of a 256-tap system with 28 non-zero coefficients.

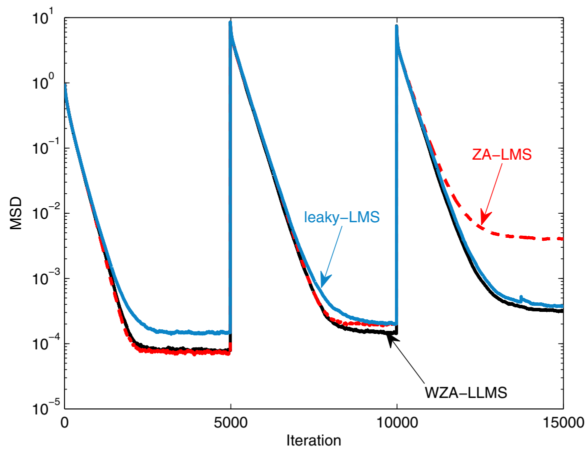

Fig. 2. Tracking and steady-state behaviors of a 16 tap adaptive filters driven by a correlated input signal.

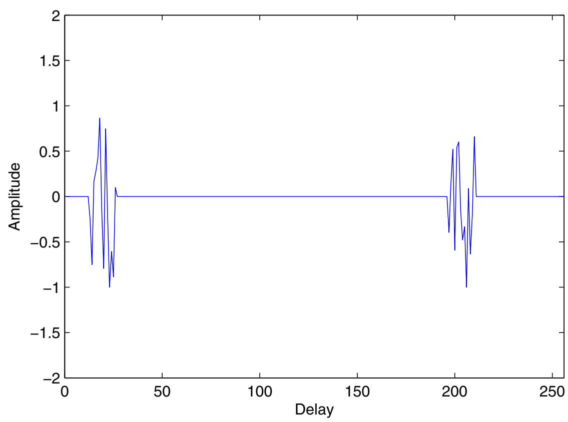

Fig. 3. Acoustic Echo Path.

Simulations are done with the following parameters: For the leaky-LMS: $\mu = 0.005$, $\gamma = 0.002$. For the WZA-LLMS: $\mu = 0.005$, $\gamma = 0.002$, $\zeta = 10$ and $\rho = 4 \times 10^{-4}$. For ZA-LMS: $\mu = 0.005$ and $\rho = 3 \times 10^{-5}$. Fig. 4 shows the average MSD estimate of all algorithms. As shown, the WZA-LLMS algorithm converges with the same rate as that of the ZA-LMS algorithm but with 2dB lower MSD. Compared to the leaky-LMS, the WZA-LLMS algorithm converges converges much faster (1000 iterations faster) with 3dB lower MSD.

IV. Conclusions

In this paper, a novel adaptive filter for sparse system identification is proposed. The proposed WZA-LLMS algorithm incorporates a log-sum penalty of the coefficients into its cost function, which resulted in a shrinkage in the update formula. This shrinkage improves the performance of the adaptive filter, especially, when the majority of coefficients are zero. The proposed WZA-LLMS algorithm is superior to the standard leaky-LMS and ZA-LMS algorithms in both convergence rate and steady-state behaviors when the system

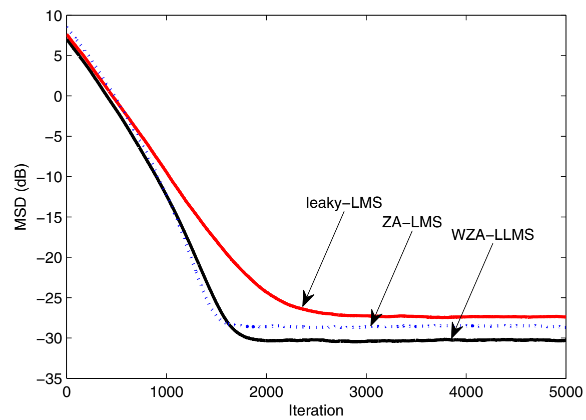

Fig. 4. Tracking and steady-state behaviors of a 256 tap acoustic echo canceller driven by a white input signal.

is sparse. Furthermore, the WZA-LLMS algorithm performs robustly even under non-sparse systems.

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