

Performance of Robust Single-User Detection in DS/CDMA Systems

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Abstract—Robust single-user detection is employed in a DS/CDMA system in which the noise process contains impulsive components. The breakdown point is computed for a mixture noise model. Noise rather than interference is shown to be the primary obstacle in achieving good performance when the signal power is low. The bit error probability expressions are also derived under a Gaussian mixture. DS/CDMA employing a robust correlator receiver performs better than the conventional matched filter in impulsive noise environment.

I. INTRODUCTION

We consider the performance advantages offered by single-user robust detectors in the presence of non-Gaussian noise. While multiple access interference (MAI) by other users has been recognized as the capacity-limiting factor in direct-sequence code division multiple access (DS/CDMA) based cellular communication systems, multi-user approaches have largely alleviated the problem when the noise process is additive Gaussian. With the solution offered by multi-user detectors, inaccurate or inappropriate noise model assumptions seem to have become the dominating issue again [7].

While multi-user detection has much to offer in the mobile-to-base station uplink, it is not considered in the standard for the downlink due to the complexity involved, the lack of resistance against adjacent cell interference, etc. Thus, the performance of single-user detectors is still of interest, especially in the presence of non-Gaussian noise. Furthermore, single-user detection is the only option if the signature sequences of other users are not known or available at the receiving base station as is the case in military applications.

In this paper, we consider the performance of robust detectors in a channel corrupted by multiple access interference and additive noise that contains occasional outliers. We investigate the breakdown point of the system, and derive probability of error expressions assuming a mixture noise model. The performance of a hard-limiter [1] and some other ad hoc nonlinear receivers [2] were investigated before, but these approaches did not contain optimization in the robust statistical sense. Throughout the paper, capital letters will denote random variables, and lower case letters will be their realizations.

II. THE DS/CDMA SIGNAL

Consider the received signal $r(t)$, where user 0 produces the desired signal, and all users employ coherent BPSK:

$$r(t) = \sqrt{2P_0}c_0(t - \tau_0)b_0(t - \tau_0)\cos(\omega_c t + \phi_0) + \sum_{m=0}^{N-1} \eta_K^m(t) + n(t). \quad (1)$$

The multiple access interference generated by the m th chips of the users $k = 1, \dots, K - 1$ is

$$\eta_K^m(t) = \sum_{k=1}^{K-1} \sqrt{2P_k}c_k^m p_{T_c}(t - mT_c - \tau_k)b_k(t - \tau_k) \times \cos(\omega_c t + \phi_k) \quad (2)$$

where P_k , b_k , $c_k^m \in \{-1, +1\}$ are the k th user's received power, bit sequence (at rate R_b) and the m th chip of the pseudo-noise (PN) sequence (at rate R_c), respectively. There are a total of K active users in the system. The noise process is denoted by $n(t)$. τ_k and ϕ_k respectively denote the time delay and phase of the k th user, which are assumed to be tracked accurately. The bits and chips are rectangular pulses of duration T_b and T_c , respectively, and p_{T_c} is the chip pulse waveform. We assume that all spreading sequences are known. In the sequel, we shall set $\tau_0 = 0$ without loss of generality. Ambient noise and MAI are statistically independent. The joint distribution of $\eta_K^m(t)$ and $n(t)$ determine the nature of the detector that has to be employed in the receiver. So far, it has been assumed that $n(t)$ is white Gaussian, and the Gaussian approximation [5] has been utilized to model $\sum_m \eta_K^m(t)$. In this paper, we consider the environment to contain impulsive elements so that $n(t)$ is still white, but no longer Gaussian.

III. ROBUST DETECTION AND THE BREAKDOWN POINT

Suppose that $n(t)$ is a stationary and memoryless noise process. Nominally, $n(t)$ is zero-mean Gaussian with variance $N_0/2$. However, occasional outliers may occur due to atmospheric disturbances, etc. Then, it is appropriate to use Huber's mixture model [4] described by the following two classes of distributions over the m th chip period:

$$\mathcal{F}_0 = \{f(x): f(x) = (1 - \epsilon_0)f_{0,\eta_K^m}(x) + \epsilon_0 f_{h,\eta_K^m}(x)\},$$

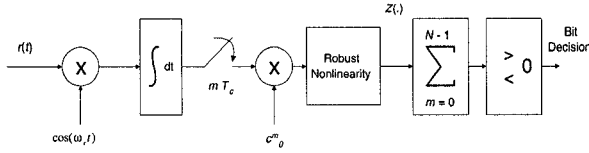


Figure 1. The robust single-user receiver for user 0.

$$\mathcal{F}_1 = \{f(x): f(x) = (1 - \epsilon_1)f_{1,\eta_K^m}(x) + \epsilon_1 f_{h,\eta_K^m}(x)\},$$

$\forall x \in \mathcal{R}$ and $h(x) \in \mathcal{H}$, where \mathcal{R} is the real line, and \mathcal{H} is the class of all one-dimensional density functions on \mathcal{R} . $f_{1,\eta_K^m}(x)$ and $f_{h,\eta_K^m}(x)$ stand for the nominal processes that generate the data alternatives +1 and -1, respectively, in the presence of MAI. Assuming that the Gaussian approximation applies to the MAI term in equation (1), f_{0,η_K^m} and f_{1,η_K^m} are both normal distributed (see Appendix). The joint probability density function (pdf) f_{h,η_K^m} covers the impulsive noise $h(x)$ and MAI. Thus, the data are corrupted by outliers that are generated by the density $h(x)$ with frequency $\epsilon_0 \in (0, 1)$ and $\epsilon_1 \in (0, 1)$ for classes \mathcal{F}_0 and \mathcal{F}_1 , respectively. The resulting outliers may be due to the impulsive background noise or MAI produced by few users when the Gaussian approximation does not hold. ϵ_0 and ϵ_1 denote the a priori probability of departure from the nominal Gaussian assumption for the respective bit types.

In this paper, we consider the receiver structure depicted in Fig. 1. In particular, a robust correlator is used where each chip is passed through a robust nonlinearity before the N chips comprising a bit are summed and forwarded to the decision device. The nonlinear processor is designed to eliminate the extreme amplitudes that occur impulsively.

In accordance with the system in Fig. 1, over one chip duration, the persistent interference component is η_K^m , described in equation (2). Invoking the Gaussian approximation, η_K^m is normal-distributed with zero mean and variance $\sigma_\eta^2 = (T_c^2/6) \sum_{k=1}^{K-1} P_k$. The nominal noise variance is $\sigma_n^2 \approx N_0 T_c/4$ for $\omega_c \gg 2/T_c$ [5]. Then, $r(t)$ is normal-distributed with mean $-\mu$ and μ , respectively under the nominal densities in Huber's classes \mathcal{F}_0 and \mathcal{F}_1 , and variance $\sigma^2 = \sigma_\eta^2 + \sigma_n^2 = (T_c^2/6) \sum_{k=1}^{K-1} P_k + N_0 T_c/4$, for sufficiently long PN code sequences and large K . The mean μ represents solely the contribution from the desired user: $\mu = \pm \sqrt{P_0/2T_c}$.

The log-likelihood ratio between the two nominal densities is $\gamma(x) = \log f_1(x)/f_0(x) = 2\mu x/\sigma^2$, where x is the output of the integrate-and-dump circuit. The starting point for determining the robust rule is the likelihood ratio of the least favorable pair (f_0^*, f_1^*) [3][4]. In particular, The objective of the robust detection rules is protection against unfavorable conditions that manifest themselves in the outlier pdf $h(x)$.

$$f_0^*(x) = \begin{cases} (1 - \epsilon_0)f_0(x) & \text{if } f_1(x)/f_0(x) < c_0 \\ (1 - \epsilon_0)f_1(x)/c_0 & \text{if } f_1(x)/f_0(x) \geq c_0, \end{cases} \quad (3)$$

$$f_1^*(x) = \begin{cases} (1 - \epsilon_1)f_1(x) & \text{if } f_1(x)/f_0(x) > c_1 \\ c_1(1 - \epsilon_1)f_0(x) & \text{if } f_1(x)/f_0(x) \leq c_1, \end{cases} \quad (4)$$

where $0 \leq c_1 < c_0 < \infty$, and (c_0, c_1) are such that f_0^* and f_1^* are legitimate density functions [3]. The likelihood ratio between the least favorable pair is

$$\gamma^*(x) = \log \frac{f_1^*(x)}{f_0^*(x)}$$

$$= \begin{cases} \log c_1 + \log \frac{1-\epsilon_1}{1-\epsilon_0} & \text{for } x \leq \frac{\sigma^2 \log c_1}{2\mu} \\ \log \frac{1-\epsilon_1}{1-\epsilon_0} + \frac{2\mu}{\sigma^2} x & \text{for } \frac{\sigma^2 \log c_1}{2\mu} < x < \frac{\sigma^2 \log c_0}{2\mu} \\ \log \frac{1-\epsilon_1}{1-\epsilon_0} + \log c_0 & \text{for } x \geq \frac{\sigma^2 \log c_0}{2\mu}. \end{cases}$$

Since each data bit is spread by N chips, a bit decision will be declared based on the block detection of all the corresponding chips at the end of T_b sec. The test function is

$$T^*(\mathbf{x}) = \frac{1}{N} \sum_{m=0}^{N-1} \log \frac{f_1^*(x_m)}{f_0^*(x_m)} \quad (5)$$

where $\mathbf{x} = \{x_0, x_1, \dots, x_{N-1}\}$. Furthermore, define

$$z(x) = \frac{\sigma^2}{2\mu} \left[\gamma^*(x) - \log \frac{1 - \epsilon_1}{1 - \epsilon_0} \right],$$

$$d_0 = \frac{\sigma^2}{2\mu} \log c_0, \quad d_1 = \frac{\sigma^2}{2\mu} \log c_1,$$

so that

$$z(x) = \begin{cases} d_1 & \text{for } x \leq d_1 \\ x & \text{for } d_1 < x < d_0 \\ d_0 & \text{for } x \geq d_0, \end{cases} \quad (6)$$

and the test in equation (5) becomes

$$T^*(\mathbf{x}) = \frac{2\mu}{N\sigma^2} \sum_{m=0}^{N-1} \left[z(x_m) + \log \frac{1 - \epsilon_1}{1 - \epsilon_0} \right].$$

The robust decision rule is

$$\delta^*(\mathbf{x}) = \begin{cases} +1 & \text{if } \frac{1}{N} \sum_{m=0}^{N-1} z(x_m) \geq 0, \\ -1 & \text{if } \frac{1}{N} \sum_{m=0}^{N-1} z(x_m) < 0, \end{cases}$$

due to the equiprobable antipodal signaling in spread spectrum systems. The thresholds d_0 and d_1 are optimized such that the average probability of error is minimized.

The least favorable pair of density functions can be determined from equations (3) and (4):

$$f_0^*(x) = \begin{cases} \frac{1-\epsilon_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x+\mu)^2}{2\sigma^2}} & \text{if } x < d_0, \\ \frac{1-\epsilon_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2} - \frac{2\mu d_0}{\sigma^2}} & \text{if } x \geq d_0. \end{cases} \quad (7)$$

$$f_1^*(x) = \begin{cases} \frac{1-\epsilon_1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} & \text{if } x > d_1, \\ \frac{1-\epsilon_1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x+\mu)^2}{2\sigma^2} + \frac{2\mu d_1}{\sigma^2}} & \text{if } x \leq d_1. \end{cases} \quad (8)$$

From equations (7) and (8) above, and by denoting the cumulative distribution function of the zero-mean, unit-variance Gaussian random variable as $\Phi(x)$, we obtain the equalities below:

$$\Phi\left(\frac{d_0 + \mu}{\sigma}\right) + e^{-\frac{2\mu d_0}{\sigma^2}} \Phi\left(\frac{-d_0 + \mu}{\sigma}\right) = \frac{1}{1 - \epsilon_0}, \quad (9)$$

$$\Phi\left(\frac{-d_1 + \mu}{\sigma}\right) + e^{\frac{2\mu d_1}{\sigma^2}} \Phi\left(\frac{d_1 + \mu}{\sigma}\right) = \frac{1}{1 - \epsilon_1}. \quad (10)$$

The set of equations (9) and (10) has no solution in (d_0, d_1) if either one of the outlier frequencies ϵ_0 or ϵ_1 exceeds 0.5, in which case the classes \mathcal{F}_0 and \mathcal{F}_1 intersect. Thus, when $\epsilon_0 > 0.5$ or $\epsilon_1 > 0.5$, the outliers dominate the environment, and no decision rule is robust enough to distinguish between the two states of the received signal.

If, on the other hand, $\epsilon = \epsilon_0 = \epsilon_1 < 0.5$, the obvious solution to equations (9) and (10) is $d_0 = -d_1$. However, at the same time, $d_1 \leq d_0$ by equation (6) implying $-d_0 = d_1 = d < 0$. Define

$$M(\epsilon, d) = (1 - \epsilon) \left[\Phi\left(\frac{-d + \mu}{\sigma}\right) + e^{\frac{2\mu d}{\sigma^2}} \Phi\left(\frac{d + \mu}{\sigma}\right) \right] = 1.$$

For a given $\epsilon < 0.5$, $M(\epsilon, d)$ increases monotonically in d , and for a given d , it decreases monotonically in ϵ . If d_ϵ^* is a solution to $M(\epsilon, d) = 1$ for a given ϵ , then $d_{\epsilon_1}^* > d_{\epsilon_2}^*$ for $\epsilon_1 > \epsilon_2$. Therefore,

$$\epsilon^* = \sup\{\epsilon: M(\epsilon, d) = 1, \text{ for a given } d\} = \min(0.5, \epsilon_0),$$

where $\epsilon_0 = 1 - [2\Phi(\mu/\sigma)]^{-1}$ is the solution for $d = 0$. Hence,

$$\epsilon^* = \min(0.5, \epsilon_0) = 1 - \left[2\Phi\left(\frac{\mu}{\sigma}\right) \right]^{-1}.$$

For $\epsilon \geq \epsilon^*$, where ϵ^* is called the breakdown point, there exists no robust detection rule and no performance guarantees can be assured. In the presence of MAI, the breakdown point is

$$\epsilon^* = 1 - \left[2\Phi\left(\sqrt{\frac{P_0 T_c}{\frac{T_c}{3} \sum_{k=1}^{K-1} P_k + \frac{N_0}{2}}}}\right) \right]^{-1}. \quad (11)$$

Equation (11) indicates that ϵ^* increases in P_0 while it decreases in P_k , $k \neq 0$, N_0 and K . Thus, the breakdown point can be raised by increasing the power of the desired signal, but the latter choice contributes to the MAI to other users. Similarly, small interferer and noise power, as well as low number of active users ensure more resistance against non-Gaussian elements in the noise process.

With power control assuring that $P_1 = P_2 = \dots = P_K$, the breakdown point is

$$= 1 - \left[2\Phi\left(\sqrt{\frac{1}{\frac{K-1}{3} + \frac{N_0}{2E_{c0}}}}\right) \right]^{-1}, \quad (12)$$

Table 1. Breakdown point against the number of active users in a DS/CDMA system with perfect power control for various E_{c0}/N_0 values.

K	$\epsilon_{I,PC}^*$	ϵ_{PC}^*				
		10 dB	0 dB	-10 dB	-20 dB	-30 dB
1	0.5000	0.4999	0.4573	0.2566	0.1011	0.0344
2	0.4782	0.4719	0.4208	0.2509	0.1008	0.0344
5	0.3802	0.3769	0.3506	0.2359	0.0999	0.0344
10	0.3038	0.3022	0.2892	0.2165	0.0985	0.0343
50	0.1634	0.1632	0.1615	0.1463	0.0890	0.0339
100	0.1214	0.1213	0.1206	0.1142	0.0804	0.0334
128	0.1088	0.1088	0.1083	0.1036	0.0765	0.0331
				E_{c0}/N_0		

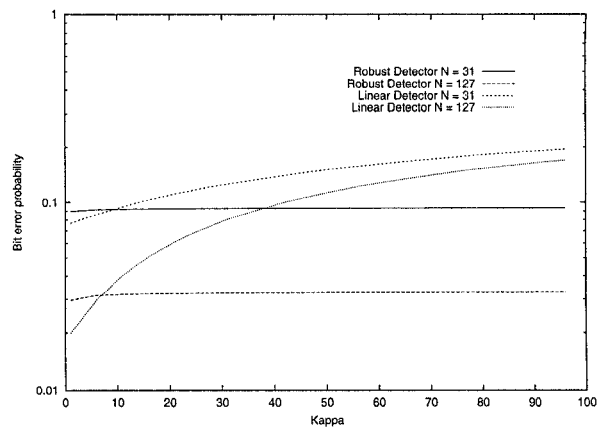


Figure 2. Bit error probability of the robust single-user correlator receiver and the matched filter against κ for $K = 20$, $R_b = 9600$ bps, $E_b/N_0 = 5$ dB and $\epsilon = 0.2$.

where E_{c0} is the received per-chip energy of user 0. If in addition to power control, the system is only interference-limited,

$$\epsilon_{I,PC}^* = 1 - \left[2\Phi\left(\sqrt{\frac{3}{K-1}}\right) \right]^{-1}. \quad (13)$$

Evaluating equations (12) and (13) with Mathematica, the results are listed in Table 1. Several important conclusions can be drawn from Table 1 for a DS/CDMA system that assures perfect power control: (1) Even when the system is only interference-limited, the breakdown point drops rapidly with increasing number of active users. (2) For sufficiently high K , the breakdown point depends very little on E_{c0}/N_0 . This is true for as low as $E_{c0}/N_0 \geq -10$ dB. For instance, $\epsilon_{I,PC}^* \approx \epsilon_{PC}^* \approx 0.105$, for $K = 128$ and $E_{c0}/N_0 \geq -10$ dB. (3) The breakdown point under ideal power control decreases to zero with increasing number of users and decreasing chip energy: $\lim_{K \rightarrow \infty} \epsilon_{PC}^* = \lim_{E_{c0}/N_0 \rightarrow -\infty} \epsilon_{PC}^* = 0$. (4) For $E_{c0}/N_0 \leq -20$ dB, the system is more noise-limited than interference-limited. Considering that the chip-energy is very low because $T_c \ll T_b$, the presence of outliers is more problematic than MAI in practical operation conditions.

Despite the fast deterioration of the breakdown point as

K grows, one should be prudent in interpreting the results. The nonlinear processing is carried out only on a chip basis, and the pessimistic outcome is partly due to the low E_{c0}/N_0 values one encounters at this stage. Furthermore the processing gain of the spread spectrum paradigm has not yet materialized. Therefore, it is more revealing to study the bit error probability that appears at the receiver output, i.e., following the summation operation, where chips are processed into bits.

IV. PROBABILITY OF ERROR FOR THE GAUSSIAN MIXTURE MODEL

Setting $\epsilon_1 = \epsilon_0 = \epsilon$ and $d_1 = -d_0 = d < 0$, the least favorable densities form a symmetric pair: $f_0^*(-x) = f_1^*(x)$. Similarly, the test function in equation (6) is now

$$z(x) = \begin{cases} d & \text{for } x \leq d \\ x & \text{for } d < x < -d \\ -d & \text{for } x \geq -d. \end{cases}$$

In the sequel, we shall suppose that $h(x) = g(x)$ is Gaussian with zero-mean and variance $\kappa\sigma_n^2$, where $\kappa \geq 1$. The outliers are generated by a Gaussian pdf with heavier tails (larger variance). Then, f_{h,η_n^m} is zero mean Gaussian with variance $\sigma_{og}^2 = \kappa\sigma_n^2 + \sigma_\eta^2$.

Define $k = d/\sigma$, $\rho = \mu/\sigma$, $k_{og} = d/\sigma_{og}$ and $k_\eta = d/\sigma_\eta$. Then, the average probability of error for a given value of κ is

$$P_e^f(\kappa) = \Phi\left(-N\frac{m_f}{v_f}\right), \quad (14)$$

where

$$m_f = E\{z(X)|f\} = (1 - \epsilon)m_{f_1},$$

$$m_{f_1} = \sigma\{\mu + [k - \mu]\Phi(k - \rho) - [k + \mu]\Phi(k + \rho) + \sigma[\phi(k - \rho) - \phi(k + \rho)]\},$$

$$v_f^2 = \text{var}\{z(X)|f\} = (1 - \epsilon)^2 v_{f_1}^2 + \epsilon^2 v_g^2 + \epsilon(1 - \epsilon)v_\eta^2,$$

$$v_{f_1}^2 = \sigma^3\{1 + \rho^2 + [k^2\sigma^{-1} - \rho^2 - 1][\Phi(k + \rho) + \Phi(k - \rho)] + [k - \rho]\phi(k + \rho) + [k + \rho]\phi(k - \rho) - m_{f_1}^2\sigma^{-3}\},$$

$$v_g^2 = \sigma_{og}^3\{1 + 2[k_{og}^2\sigma_{og}^{-1} - 1]\Phi(k_{og}) + 2k_{og}\phi(k_{og})\},$$

$$v_\eta^2 = \sigma_\eta^3\{1 + 2[k_\eta^2\sigma_\eta^{-1} - 1]\Phi(k_\eta) + 2k_\eta\phi(k_\eta)\},$$

and $\phi(x)$ is the pdf of the zero-mean, unit-variance normal random variable.

V. RESULTS

A. Numerical Evaluations

The average probability of error for the matched filter is as in equation (14) with the following substitutions for m_f and v_f^2 :

$$m_f = E\{X|f\} = (1 - \epsilon)\mu$$

$$v_f^2 = (1 - \epsilon)^2(\sigma_n^2 + \sigma_\eta^2) + 2\epsilon(1 - \epsilon)\sigma_\eta^2 + \epsilon^2(\kappa\sigma_n^2 + \sigma_\eta^2).$$

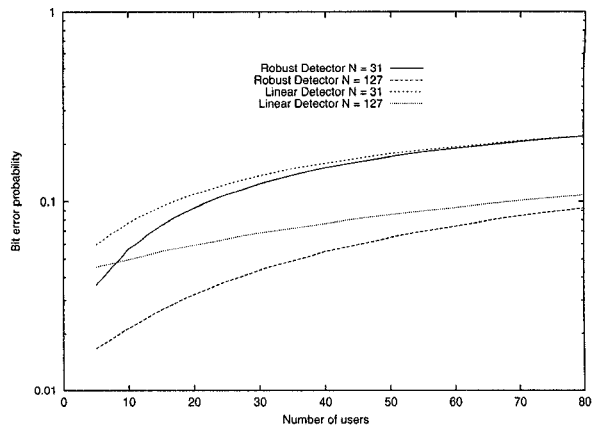


Figure 3. Bit error probability of the robust single-user receiver and the matched filter against the number of active users ($R_b = 9600$ bps, $E_b/N_0 = 5$ dB, $\kappa = 20$, $\epsilon = 0.2$).

The bit error probability expressions for the robust receiver and the matched filter are evaluated via equation (14) for $R_b = 9600$ bps, $E_b/N_0 = 5$ dB and $\epsilon = 0.2$. The processing gain values of $N = 127$ and $N = 31$, with the respective optimal thresholds of $d = -0.00036$ and $d = -0.00086$ (setting N_0 to unity), are considered. For 20 active users, Fig. 2 indicates that the matched filter with $N = 31$ ($N = 127$) performs slightly better when the noise process is near-Gaussian for $\kappa \leq 10$ ($\kappa \leq 7$). However, the robust structure offers significant gains when impulsive elements begin to dominate. In fact, $P_e^f(\kappa) \approx 3.3 \times 10^{-2}$ for the robust single-user detector when $N = 127$ and $\kappa > 10$. The margin of improvement becomes larger with increased N . Once the trimming threshold d is set, it does not matter how large the outlier amplitudes are, and hence we have the bit error probability independent of κ .

Fig. 3 shows that for relatively few users, the impulsive noise is the predominant disturbance rather than MAI, for sufficiently high processing gain. When $N = 31$ and the number of active users exceeds 40, the advantages offered by the robust receiver disappear unless the higher processing gain is employed.

In Fig. 4, for a fixed number of users ($K = 10$) and $\epsilon = 0.2$, the bit error probabilities of the robust and the linear receivers in impulsive channel are shown. When the channel is severely impulsive ($\kappa = 100$), the performance of the linear receiver is degraded relative to the robust receiver. The bit error probability of the matched filter increases in κ , while the robust receiver effectively eliminates outliers and provides relatively low error rates for a range of E_b/N_0 values.

B. Simulations

Computer simulations are used to validate the analytical results, and to determine the bit error rates of the robust and linear receivers in various channel conditions. The simulations are run to generate 1×10^5 information

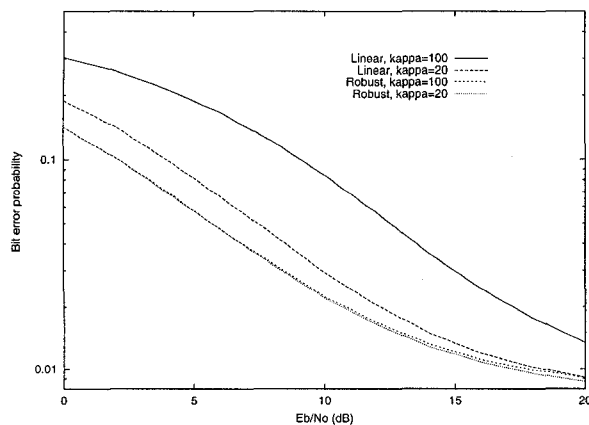


Figure 4 Bit error probability of the robust single-user receiver and the matched filter against E_b/N_0 ($R_b = 9600$ bps, $K = 10$, $\epsilon = 0.2$, $N = 31$).

bits at $E_b/N_0 = 5$ dB. The spreading sequence of each user is a shifted version of an m -sequence of length $N = 31$. Fig. 5 indicates that the analysis and simulation results are in good agreement. The performance of the robust detector is superior to that of the linear detector in impulsive channel with parameters $\kappa = 20$ and $\epsilon = 0.2$. The difference in performance decreases with increasing number of users. There is a degradation in the performance of the robust receiver relative to the linear receiver in Gaussian channel. The result is expected since the matched filter is the optimal detector in Gaussian noise.

VI. CONCLUSION

The breakdown point performance of the robust single-user DS/CDMA receiver is discouraging due to the low per-chip energy. This result resembles the typical pre-detection performance of a spread spectrum system. Indeed, the robust correlator detector enjoys much lower bit error probability over the conventional linear receiver in impulsive channel. When noise is Gaussian, the robust procedure suffers a marginal performance deterioration.

VII. APPENDIX

Below is the sketch of the proof of the asymptotic variance of η_K^m . The proof closely follows Appendix C in [6].

After downconversion to the baseband, despreading and integration over one chip duration,

$$\eta_k^m = \sqrt{\frac{P_k}{2}} \cos \phi_k \{X_k \Delta_k + Y_k (T_c - \Delta_k)\}$$

where X_k and Y_k denote respectively the product of c_0^i with c_k^j and c_k^{j+1} , for some i, j , due to the lack of synchronization. The former overlap is of duration Δ_k while the latter is $T_c - \Delta_k$. Thus, both X_k and Y_k are equiprobable binomial: $X_k, Y_k \in \{-1, +1\}$. Conditioning on $\{\Delta_k\}_{k=1}^{K-1}$, $\{\phi_k\}_{k=1}^{K-1}$ and $\{P_k\}_{k=1}^{K-1}$,

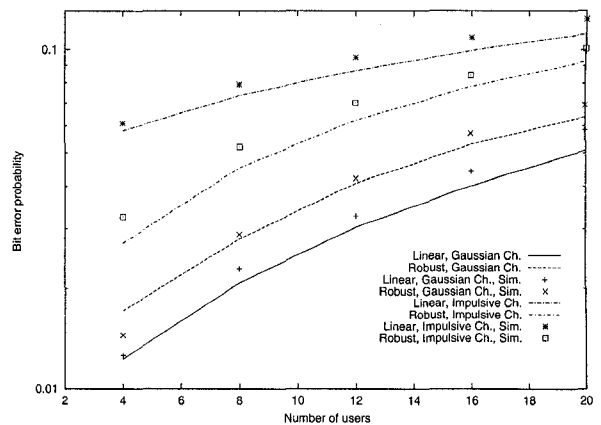


Figure 5 Bit error probability of the robust single-user receiver and the matched filter against the number of active users ($R_b = 9600$ bps, $E_b/N_0 = 5$ dB, $\epsilon = 0.2$, $\kappa = 20$, $N = 31$).

and assuming uncorrelated interferers [6]

$$\begin{aligned} \text{var}\{\eta_K^m | \{\Delta_k\}, \{\phi_k\}, \{P_k\}\} &= \sum_{k=1}^{K-1} E[(\eta_k^m)^2 | \{\Delta_k\}, \{\phi_k\}, \{P_k\}] \\ &= \sum_{k=1}^{K-1} \frac{T_c^2 P_k}{2} \cos^2 \phi_k E[W_k^2 | \{\Delta_k\}, \{\phi_k\}, \{P_k\}] \end{aligned}$$

where $W_k = X_k \Delta_k + Y_k (T_c - \Delta_k)$, and X_k and Y_k are uncorrelated.

$$E[W_k^2 | \{\Delta_k\}, \{\phi_k\}, \{P_k\}] = \frac{P_k}{2} \cos^2 \phi_k \{T_c^2 - 2T_c \Delta_k + 2\Delta_k^2\}.$$

Taking the expectation over $\{\phi_k\}_{k=1}^{K-1}$, which are uniformly distributed in $[0, 2\pi)$,

$$\text{var}\{\eta_K^m | \{\Delta_k\}, \{P_k\}\} = \sum_{k=1}^{K-1} \frac{P_k}{4} \{T_c^2 - 2T_c \Delta_k + 2\Delta_k^2\}.$$

Finally, computing the expectation over $\{\Delta_k\}_{k=1}^{K-1}$ which are assumed to be uniform in $[0, T_c)$, $\text{var}\{\eta_K^m | \{P_k\}\} = T_c^2 P_k / 6$.

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