

Chapter 4

PASSIVE FILTER DESIGN

4.1 Introduction

This chapter introduces the basic principles on filter design. After basic definition and terminologies for resonant circuits are reviewed, the procedure to design a low-pass prototype filter using passive elements to meet desired specifications is discussed. Given a low-pass prototype, all other filter configurations, including high-pass, band-pass and band-stop can be readily implemented.

4.2 Resonant Circuits [1]

4.2.1 Basic Definitions

Figure 4.1 shows a typical frequency response of a resonant circuit. Basic terminologies for the filter are given as follows.

The bandwidth BW is the difference between the upper and the lower frequencies ($f_2 - f_1$) at which the amplitude is -3 dB below the amplitude at the resonant frequency f_0 .

The quality factor Q is a measure of the selectivity of the filter and is defined as

$$Q = \frac{f_0}{BW} = \frac{f_0}{f_2 - f_1} \quad (4.1)$$

The insertion loss measures the loss in the amplitude due to the filter and is the difference between the actual amplitude and the ideal amplitude at the resonant frequency.

The ripple is a measure of the flatness of the filter in the passband and is determined as the difference between the maximum and the minimum amplitude in the passband.

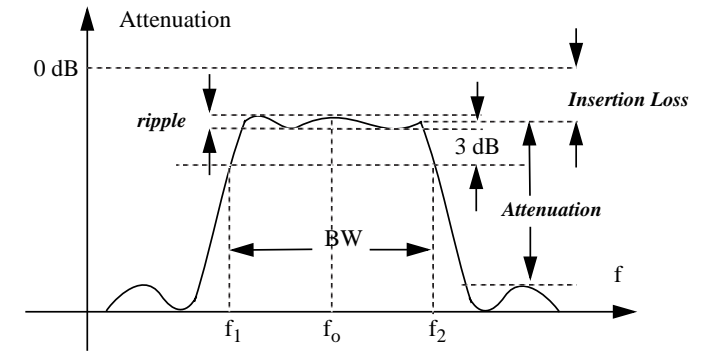


Fig. 4.1 Typical frequency response of a filter

4.2.2 Quality Factor

The selectivity of a filter is measured by its quality factor Q and depends not only on the source and the load resistance but also on the individual quality factors of the inductor and the capacitor.

Consider a general resonant circuit shown in Fig. 4.2 with the inductor and the capacitor being lossless, the quality factor Q is given by

$$Q = \frac{R_{PT}}{X_P} \quad (4.2)$$

where R_{PT} is the equivalent parallel resistance,

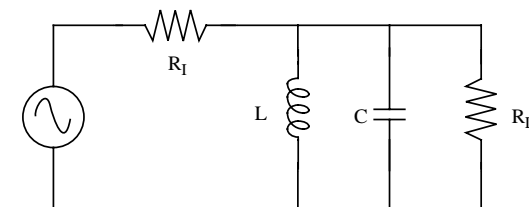


Fig. 4.2 Schematic of a simple resonant circuit with source and load resistors

$$R_{pT} = R_1 \parallel R_L \quad (4.3)$$

and X_p is either the inductive or capacitive reactance, which are equal at the resonant frequency,

$$X_p = \omega L = \frac{1}{\omega C} \quad (4.4)$$

If the inductor or the capacitor is lossy, a series resistor R_s can be included, and this series resistor can be transformed into an equivalent parallel resistor using the following transformation

$$R_p = R_s(1 + Q_o^2) \quad (4.5)$$

$$L_p = L_s \left(1 + \frac{1}{Q_o^2}\right) \quad (4.6)$$

$$C_p = \frac{C_s}{1 + \frac{1}{Q_o^2}} \quad (4.7)$$

where Q_o is the quality factor of the component. Figure 4.3 illustrates the relationships for the components before and after transformation.

As a review, in terms of the series resistance, the individual quality factors of an inductor and a capacitor are given as

$$Q_L = \frac{X_L}{R_s} = \frac{\omega L}{R_s} \quad (4.8)$$

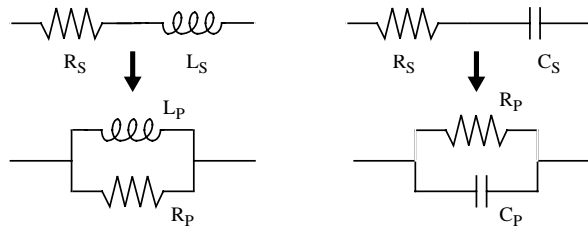


Fig. 4.3 Series-to-parallel transformation for lossy inductors and capacitors

$$Q_c = \frac{X_L}{R_s} = \frac{1}{\omega C R_s} \quad (4.9)$$

After the series resistor is transformed into an equivalent parallel resistor, the parallel resistance can be included in Eq. 4.3 to calculate the total equivalent resistance R_{pT} , and Eq. 4.2 can then be used to calculate the overall quality factor for the resonant circuit.

4.2.3 Coupling of Resonant Circuits

In many applications, a single resonant circuit may not be enough to provide desirable attenuation. One possible solution is to couple two or more resonant circuits to each other using either capacitor, inductor, or active devices.

4.2.3.1 Capacitive Coupling

Figure 4.4 illustrates how two resonant circuits can be coupled using a coupling capacitor C_c . Affect of the coupling capacitor on the total response of the resonant circuits is shown in Fig. 4.5. Typically, the value of the coupling capacitor has to be properly chosen to achieve a critical coupling. If the capacitor is too large, the coupling is too large, and the response has a large bandwidth with two resonant peaks. On the other hand, if the capacitor is too small, the coupling is not enough, and the total insertion loss can be unacceptably high.

For a critical coupling, the value of the coupling capacitor is given by

$$C_c = \frac{C}{Q} \quad (4.10)$$

where C and Q are the capacitor and the quality factor of a single resonator, respectively.

It is important to note that the coupling will increase the attenuation slope but at the same time increase the bandwidth of the total response. As an example, for two critically-coupled identical

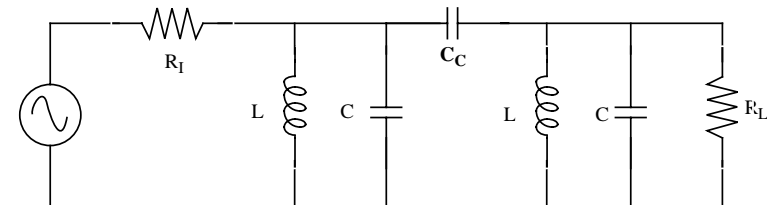


Fig. 4.4 Capacitive coupling of two resonant circuits

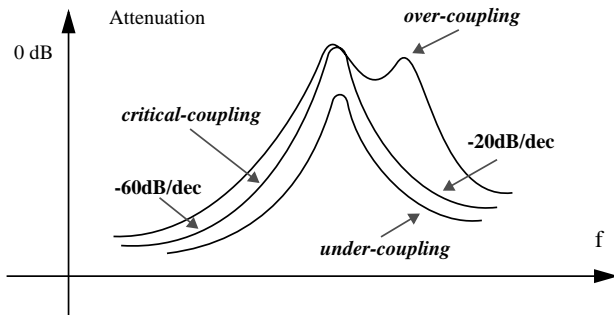


Fig. 4.5 Affect of capacitive coupling on the response of the resonator

resonant circuits, the relationship between the total quality factor Q_T and the quality factor Q of individual resonant circuit can be approximated as

$$Q_T = 0.71Q \tag{4.11}$$

In addition, the response is not symmetrical. At low frequencies, the two shunt capacitors can be neglected, which results in an equivalent circuit with three reactive components, and thus the attenuation slope is approximately -60 dB/dec. At high frequencies, the two inductors become negligible, which results in an equivalent circuit with three capacitors in a loop or equivalently a single shunt capacitor, and consequently the attenuation slope becomes -20 dB/dec.

4.2.3.2 Inductive Coupling

Instead of using a capacitor, two resonant circuits can also be coupled using a coupling inductor L_C as illustrated in Fig. 4.6. Similar to capacitive coupling, the overall response can be

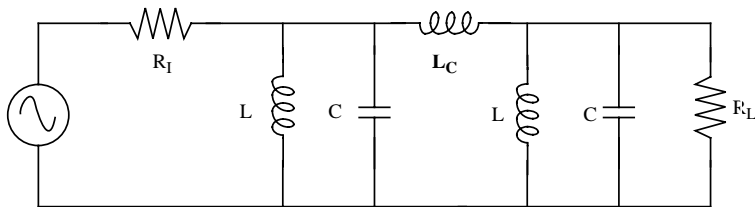


Fig. 4.6 Inductive coupling of two resonant circuits

over-coupling, under-coupling or critical coupling depending on the value of the coupling inductor. To achieve a critical coupling, the value of the coupling inductor is given by

$$L_c = QL \tag{4.12}$$

where L and Q are the inductor and the quality factor of a single resonator, respectively.

Interestingly, the overall response using an inductive coupling is not only symmetrical but is exactly the mirror of the response using a capacitive coupling. More specifically, at low frequencies, the two shunt capacitors can be neglected, which results in an equivalent circuit with three inductors in a loop, and thus the attenuation slope is approximately -20 dB/dec. At high frequencies, the two inductors become negligible, which results in an equivalent circuit with three reactive components and an attenuation slope of -60 dB/dec.

4.2.3.3 Active Coupling

Resonant circuits can be coupled to each other using active devices instead of passive devices as shown in Fig. 4.7. Unlike capacitive and inductive coupling, in which the overall bandwidth of the cascaded resonant circuits becomes larger, the overall bandwidth with active coupling is actually narrower. In addition, as opposed to an insertion loss, a gain can be achieved in the passband.

For a cascade of a number of identical stages, it can be proved that the overall quality factor Q_T in terms of the individual quality factor Q and the number of stages n is given by

$$Q_T = \frac{Q}{\sqrt{2^{1/n} - 1}} \tag{4.13}$$

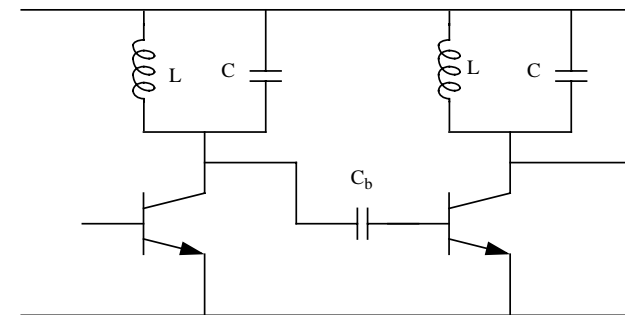


Fig. 4.7 Coupling of two resonant circuits using active devices

4.3 Passive Filter Design [1]

The resonant circuits considered in the previous sections can only implement a bandpass response. For many other applications, such as low-pass, high-pass, or even band-stop filters, other implementations are required.

This section will first introduce the concept of normalization and scaling to standardize all the filter designs based on low-pass prototypes. These low-pass prototypes can be implemented using either of the two topologies shown in Fig. 4.8, in both of which the order of the filter is determined by the number of the reactive components. The question is how to design the order and the values of the components to meet some response and attenuation characteristics of the desired filter.

After a review of different types of filters, a detailed design procedure will be described, and examples will be presented as illustration.

4.3.1 Frequency and Impedance Normalization and Scaling

To be able to present all the filter design and performance in a compact and useful way, the concept of normalization and scaling are normally used in filter design.

Basically, through normalization, all the filters - whether they are low-pass, high-pass, band-pass or even band-stop - will be generated based on appropriate low-pass prototypes that are normalized to a cut-off frequency of 1 rad/s = 0.159 Hz and for a load resistor of 1 Ω.

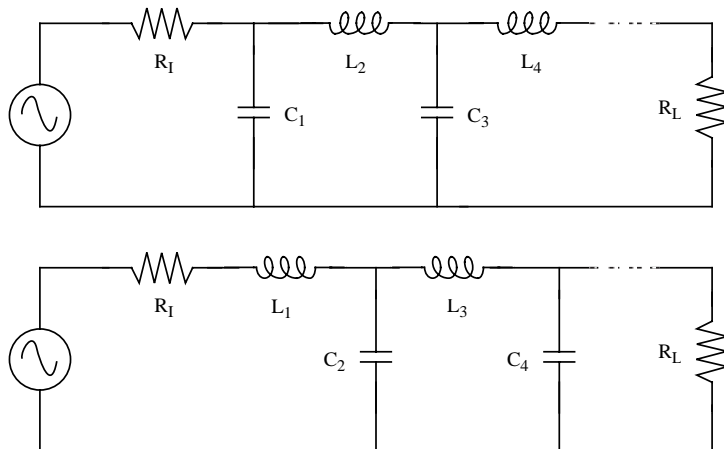


Fig. 4.8 The two topologies to realize low-pass prototypes

After suitable components with normalized values have been designed from appropriate low-pass prototypes, scaling will be applied to achieve desirable frequencies and impedances. The formulas used to scale the normalized component values (R_{Sn} , C_n , L_n) to their final values (R_{Sf} , C_f , L_f) are as follows

$$C_f = \frac{C_n}{2\pi f_c R_{L_f}} \tag{4.14}$$

$$L_f = \frac{R_{L_f} L_n}{2\pi f_c} \tag{4.15}$$

$$R_{S_f} = R_{S_n} R_{L_f} \tag{4.16}$$

where R_{L_f} is the final desirable value of the load resistor.

Note that because that the load resistor was normalized to a value of 1 Ω to begin with, its final value is actually the desired value R_{L_f} without a need of any scaling.

4.3.2 Filter Types

We will review and consider the three basic filter types: Butterworth, Chebyshev, and Bessel. Depending on the filter specification, one filter type may be preferred to the others.

4.3.2.1 Butterworth Filter

Butterworth filters are low-Q filters that are most frequently used to obtain a filter response with no ripple, flat passband, and reasonable attenuation slope.

The attenuation of a Butterworth filter is given by

$$A_{dB}(\omega) = 10 \log \left[1 + \left(\frac{\omega}{\omega_c} \right)^n \right] \tag{4.17}$$

where ω_c is the cut-off frequency and n is the order of the filter.

Figure 4.9 shows typical attenuation curves for a Butterworth low-pass prototype. These curves are quite useful in providing quickly the minimum order required to achieve a specified attenuation.

Once the order of the filter is determined, the normalized values for all the components shown in Fig. 4.8 are well defined. For Butterworth low-pass filters with source and load resistors being 1 Ω each, the reactance of the k-th element in the ladder is given by

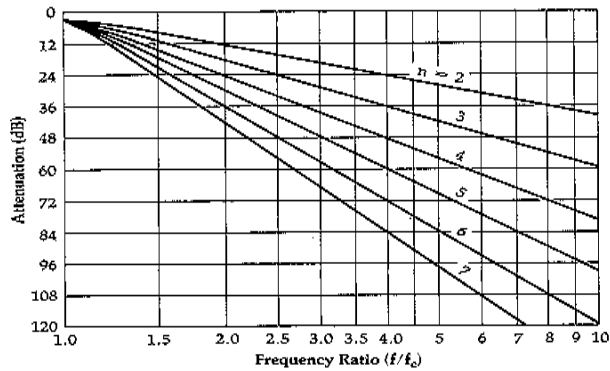


Fig. 4.9 Attenuation curves for Butterworth low-pass prototype

$$X_k = 2 \sin \frac{(2k-1)\pi}{2n} \tag{4.18}$$

For Butterworth low-pass prototypes with different source and load resistors, the required component values are available for look up from tables. As an example, Table 4.1 lists the normalized values for Butterworth low-pass prototypes of order 2 to 4 for different ratios of the load and source resistors. Note that the top circuit topology corresponds to a given ratio of R_S/R_L whereas the bottom topology is used for a given ratio of R_L/R_S .

4.3.2.2 Chebyshev Filter

Chebyshev filters are in general better than Butterworth because of its larger attenuation. However, there will exist non-zero ripple in the passband.

The attenuation of a Chebyshev filter is given by

$$A_{dB}(\omega) = 10 \log \left[1 + \varepsilon^2 C_n^2 \left(\frac{K\omega}{\omega_c} \right) \right] \tag{4.19}$$

where ω_c is the cut-off frequency, n is the order of the filter, $C_n(K\omega/\omega_c)$ is the Chebyshev polynomial to the order n evaluated at a frequency of $K\omega/\omega_c$, and ε and K are constants defined as

$$\varepsilon = \sqrt{10^{R_{dB}/10} - 1} \tag{4.20}$$

n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.111	1.035	1.835		
	1.250	0.949	2.121		
	1.429	0.897	2.439		
	1.667	0.866	2.828		
	2.000	0.448	3.346		
	2.500	0.342	4.095		
	3.333	0.245	5.313		
	5.000	0.156	7.707		
	10.000	0.074	14.814		
	∞		1.414	0.707	
3	0.900	0.806	1.633	1.599	
	0.800	0.844	1.384	1.926	
	0.700	0.915	1.165	2.277	
	0.600	1.023	0.985	2.702	
	0.500	1.181	0.779	3.261	
	0.400	1.425	0.604	4.064	
	0.300	1.838	0.440	5.363	
	0.200	2.669	0.284	7.910	
	0.100	5.167	0.138	15.435	
	∞	1.500	1.333	0.500	
4	1.111	0.466	1.592	1.744	1.469
	1.250	0.388	1.695	1.511	1.811
	1.429	0.325	1.862	1.291	2.175
	1.667	0.269	2.103	1.082	2.613
	2.000	0.218	2.452	0.883	3.187
	2.500	0.169	2.986	0.691	4.009
	3.333	0.124	3.883	0.507	5.338
	5.000	0.080	5.684	0.331	7.940
	10.000	0.039	11.094	0.162	15.642
	∞	1.531	1.577	1.082	0.383

n	R_L/R_S	L_1	C_2	L_3	C_4
2	1.111	1.035	1.835		
	1.250	0.949	2.121		
	1.429	0.897	2.439		
	1.667	0.866	2.828		
	2.000	0.448	3.346		
	2.500	0.342	4.095		
	3.333	0.245	5.313		
	5.000	0.156	7.707		
	10.000	0.074	14.814		
	∞		1.414	0.707	
3	0.900	0.806	1.633	1.599	
	0.800	0.844	1.384	1.926	
	0.700	0.915	1.165	2.277	
	0.600	1.023	0.985	2.702	
	0.500	1.181	0.779	3.261	
	0.400	1.425	0.604	4.064	
	0.300	1.838	0.440	5.363	
	0.200	2.669	0.284	7.910	
	0.100	5.167	0.138	15.435	
	∞	1.500	1.333	0.500	
4	1.111	0.466	1.592	1.744	1.469
	1.250	0.388	1.695	1.511	1.811
	1.429	0.325	1.862	1.291	2.175
	1.667	0.269	2.103	1.082	2.613
	2.000	0.218	2.452	0.883	3.187
	2.500	0.169	2.986	0.691	4.009
	3.333	0.124	3.883	0.507	5.338
	5.000	0.080	5.684	0.331	7.940
	10.000	0.039	11.094	0.162	15.642
	∞	1.531	1.577	1.082	0.383

Table 4.1 Component values for Butterworth low-pass prototype

$$K = \cosh B = \cosh \left\{ \frac{1}{n} \left[\operatorname{acosh} \left(\frac{1}{\varepsilon} \right) \right] \right\} \tag{4.21}$$

with R_{dB} being the ripple in dB, \cosh and acosh being the hyperbolic cosine and inverse hyperbolic cosine functions, respectively. It may be helpful to be reminded that

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{4.22}$$

$$\text{acosh}(x) = \ln(x \pm \sqrt{x^2 - 1}) \tag{4.23}$$

As reference, the Chebyshev polynomials for the order n being 1 to 7 are listed in Table 4.2 and can be used to estimate the attenuation of the filter response using the above equations.

However, it is typically much more convenient to use attenuation curves to estimate the attenuation for a given filter order or the minimum order for some specified attenuation. The attenuation curves for Chebyshev low-pass filters with 0.01-dB and 0.1-dB ripples are plotted in Figs. 4.10 and 4.11, respectively. Table 4.3 lists the normalized values for Chebyshev low-pass prototypes for 0.1-dB ripple of order 2 to 4 for different ratios of the load and source resistors.

n	Chebyshev Polynomial
1	$\frac{\omega}{\omega_c}$
2	$2\left(\frac{\omega}{\omega_c}\right)^2 - 1$
3	$4\left(\frac{\omega}{\omega_c}\right)^3 - 3\left(\frac{\omega}{\omega_c}\right)$
4	$8\left(\frac{\omega}{\omega_c}\right)^4 - 8\left(\frac{\omega}{\omega_c}\right)^2 + 1$
5	$16\left(\frac{\omega}{\omega_c}\right)^5 - 20\left(\frac{\omega}{\omega_c}\right)^3 + 5\left(\frac{\omega}{\omega_c}\right)$
6	$32\left(\frac{\omega}{\omega_c}\right)^6 - 48\left(\frac{\omega}{\omega_c}\right)^4 + 18\left(\frac{\omega}{\omega_c}\right)^2 - 1$
7	$64\left(\frac{\omega}{\omega_c}\right)^7 - 112\left(\frac{\omega}{\omega_c}\right)^5 + 56\left(\frac{\omega}{\omega_c}\right)^3 - 7\left(\frac{\omega}{\omega_c}\right)$

Table 4.2 Chebyshev polynomials to the order n

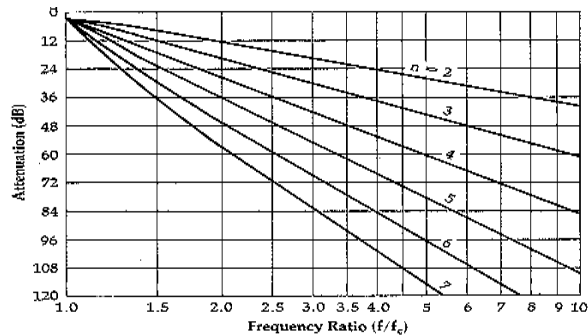


Fig. 4.10 Attenuation curves for Chebyshev low-pass prototype with 0.01-dB ripple

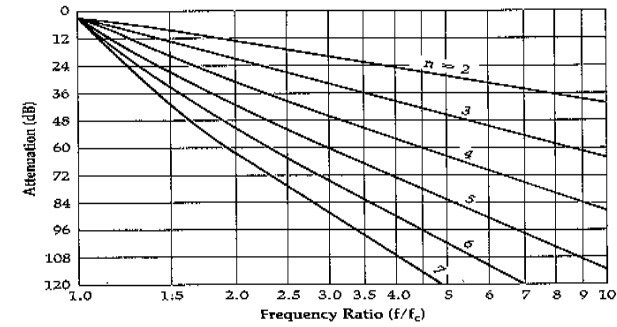


Fig. 4.11 Attenuation curves for Chebyshev low-pass prototype with 0.1-dB ripple

n	R_B/R_L	C_1	L_2	C_3	L_4
2	1.355	1.209	1.638		
	1.429	0.977	1.982		
	1.667	0.733	2.439		
	2.000	0.560	3.054		
	2.500	0.417	3.827		
	3.333	0.293	5.050		
	5.000	0.184	7.426		
10.000	0.087	14.433			
∞	1.391	0.819			
3	1.000	1.433	1.594	1.433	
	0.900	1.426	1.494	1.622	
	0.800	1.451	1.356	1.671	
	0.700	1.521	1.193	2.190	
	0.600	1.648	1.017	2.603	
	0.500	1.853	0.838	3.159	
	0.400	2.186	0.660	3.968	
	0.300	2.763	0.486	5.279	
	0.200	3.942	0.317	7.550	
	0.100	7.512	0.155	15.466	
∞	1.513	1.510	0.716		
4	1.355	0.992	2.146	1.585	1.341
	1.429	0.779	2.348	1.429	1.700
	1.667	0.576	2.730	1.185	2.243
	2.000	0.440	3.227	0.967	2.856
	2.500	0.329	3.961	0.760	3.698
	3.333	0.233	5.178	0.590	5.030
	5.000	0.148	7.607	0.367	7.614
10.000	0.070	14.887	0.180	15.230	
∞	1.511	1.768	1.455	0.873	

n	R_L/R_S	L_1	C_2	L_3	C_4
2	1.355	1.209	1.638		
	1.429	0.977	1.982		
	1.667	0.733	2.439		
	2.000	0.560	3.054		
	2.500	0.417	3.827		
	3.333	0.293	5.050		
	5.000	0.184	7.426		
10.000	0.087	14.433			
∞	1.391	0.819			
3	1.000	1.433	1.594	1.433	
	0.900	1.426	1.494	1.622	
	0.800	1.451	1.356	1.671	
	0.700	1.521	1.193	2.190	
	0.600	1.648	1.017	2.603	
	0.500	1.853	0.838	3.159	
	0.400	2.186	0.660	3.968	
	0.300	2.763	0.486	5.279	
	0.200	3.942	0.317	7.550	
	0.100	7.512	0.155	15.466	
∞	1.513	1.510	0.716		
4	1.355	0.992	2.146	1.585	1.341
	1.429	0.779	2.348	1.429	1.700
	1.667	0.576	2.730	1.185	2.243
	2.000	0.440	3.227	0.967	2.856
	2.500	0.329	3.961	0.760	3.698
	3.333	0.233	5.178	0.590	5.030
	5.000	0.148	7.607	0.367	7.614
10.000	0.070	14.887	0.180	15.230	
∞	1.511	1.768	1.455	0.873	

Table 4.3 Component values for Chebyshev low-pass prototype for 0.1-dB ripple

4.3.2.3 Bessel Filter

Bessel filters are in general more preferred to Butterworth and Chebyshev because of their constant group delay or linear phase response. However, the attenuation is not as good.

For frequencies $\omega < 2\omega_c$, the attenuation of a Bessel filter can be approximated by

$$A_{dB}(\omega) = 3 \left(\frac{\omega}{\omega_c} \right)^2 \tag{4.24}$$

For frequencies $\omega > 2\omega_c$, the attenuation of a Bessel filter can be estimated as 20 dB per decade.

The attenuation curves that can be conveniently used for design Bessel low-pass prototypes are shown in Fig. 4.12. The normalized values for the components in a Bessel low-pass filter are listed in Table 4.4.

4.3.3 Design Procedure for a Low-Pass Filter

The following is a step-by-step procedure how to design a low-pass filter.

- a. Define or specify the response in terms of
 - Desirable attenuation at frequencies of interest
 - Maximum allowable ripple in the passband
 - Ratio of the source resistance R_S and the load resistance R_L
- b. Normalize the frequencies of interest in terms of the cut-off frequency f_c to determine the

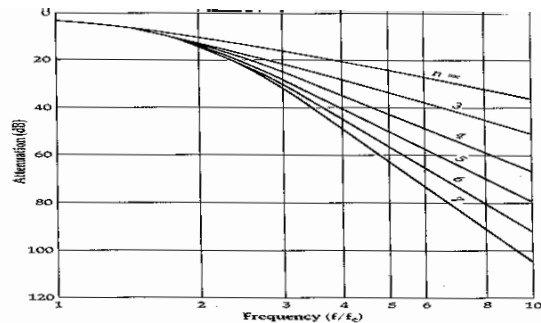


Fig. 4.12 Attenuation curves for Bessel low-pass prototypes

n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.000	0.576	2.148		
	1.111	0.508	2.310		
	1.250	0.443	2.510		
	1.429	0.390	2.764		
	1.667	0.319	3.099		
	2.000	0.260	3.565		
	2.500	0.203	4.258		
	3.333	0.149	5.405		
	5.000	0.097	7.688		
	10.000	0.047	14.510		
∞		1.362	0.454		
3	1.000	0.337	0.971	2.203	
	0.900	0.371	0.865	2.375	
	0.800	0.412	0.761	2.587	
	0.700	0.466	0.658	2.858	
	0.600	0.537	0.558	3.216	
	0.500	0.635	0.459	3.714	
	0.400	0.783	0.362	4.457	
	0.300	1.028	0.267	5.689	
	0.200	1.518	0.175	8.140	
	0.100	2.983	0.096	15.470	
∞		1.463	0.843	0.293	
4	1.000	0.233	0.673	1.082	2.240
	1.111	0.209	0.742	0.967	2.414
	1.250	0.184	0.829	0.853	2.630
	1.429	0.160	0.941	0.741	2.907
	1.667	0.136	1.089	0.630	3.273
	2.000	0.112	1.295	0.520	3.782
	2.500	0.089	1.504	0.412	4.543
	3.333	0.066	2.117	0.306	5.805
	5.000	0.043	3.142	0.201	8.319
	10.000	0.021	6.209	0.099	15.837
∞		1.501	0.978	0.613	0.211
n	R_L/R_S	L_1	C_2	L_3	C_4

Table 4.4 Normalized component values for Bessel low-pass prototype

ratios f/f_c

- c. Determine the appropriate filter type and the minimum order to meet the required specification using either attenuation equations or curves
- d. Determine the low-pass prototype values of the components
- e. Scale all the elements to the desired frequency and impedance using the formulas given in Eqs. 4.14 - 4.16

4.3.4 Design Procedure for a High-Pass Filter

The following is a step-by-step procedure how to design a high-pass filter based on a low-pass prototype.

- a. Define or specify the response in terms of
 - Desirable attenuation at frequencies of interest
 - Maximum allowable ripple in the passband
 - Ratio of the source resistance R_S and the load resistance R_L
- b. Normalize the frequencies of interest in terms of the cut-off frequency f_c to determine the ratios f_c/f
- c. Determine the appropriate filter type and the minimum order to meet the required specification using either attenuation equations or curves for low-pass prototypes using the ratios f_c/f (as opposed to the ratios f/f_c)
- d. Determine the low-pass prototype values of the components
- e. Replace each reactive component in the low-pass prototype with its dual and with a reciprocal value. In other words, every inductor with a value of L_n is replaced by a capacitor of a value C_n given by

$$C_n = \frac{1}{L_n} \quad (4.25)$$

and every capacitor with a value of C_n is replaced by an inductor of a value L_n given by

$$L_n = \frac{1}{C_n} \quad (4.26)$$

- f. Scale all the elements to the desired frequency and impedance using the formulas given in Eqs. 4.14 - 4.16

4.3.5 Design Procedure for a Band-Pass Filter

To transform a low-pass prototype to a band-pass response, the ratio of the bandwidths becomes important rather than the ratio of the frequencies as in the low-pass case, as indicated in Fig. 4.13. However, for the same attenuation characteristics, the attenuation curves for low-pass prototypes can be readily used to determine the order and the attenuation of a band-pass filter as long as the normalized ratio f/f_c is replaced by BW/BW_c .

More specifically, a step-by-step procedure on how to design a band-pass filter based on a low-pass prototype can be outlined as follows.

- a. Define or specify the response in terms of
 - Desirable attenuation at frequencies of interest
 - Maximum allowable ripple in the passband
 - Ratio of the source resistance R_S and the load resistance R_L

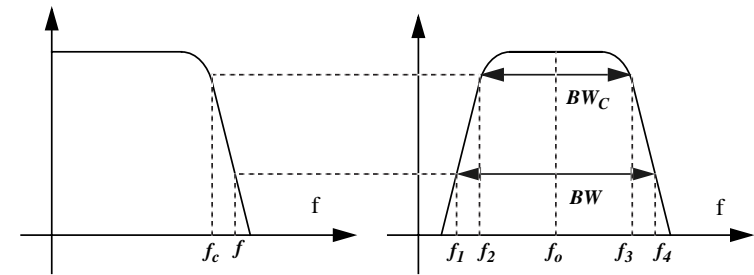


Fig. 4.13 Ratio of frequencies in a low-pass filter and ratio of bandwidths in a band-pass

- b. Normalize the bandwidth of interest BW in terms of the cut-off bandwidth BW_c to determine the ratios BW/BW_c
- c. Determine the appropriate filter type and the minimum order to meet the required specification using either attenuation equations or curves for low-pass prototypes using the ratios BW/BW_c (as opposed to the ratios f/f_c)
- d. Determine the low-pass prototype values of the components
- e. Replace each inductor in the low-pass prototype with a series resonant circuit and each capacitor with a parallel resonant circuit as illustrated in Fig. 4.14. Note that both elements in each of the resonant circuits have the same normalized value.
- f. Scale all the elements to the desired frequency and impedance as follows.

For *parallel* resonant components,

$$C_{pf} = \frac{C_p}{2\pi(BW_c)R_L} \quad (4.27)$$

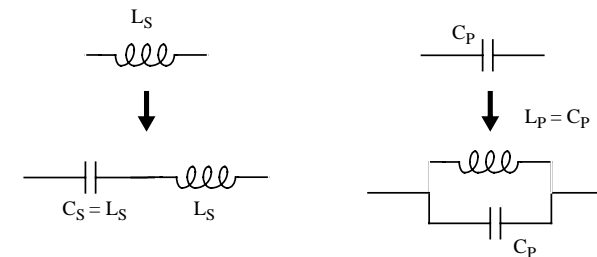


Fig. 4.14 Transformation of components from low-pass to band-pass filter

$$L_{pf} = \frac{R_L(BW_c)}{2\pi f_o^2 L_p} \quad (4.28)$$

For *series* resonant components,

$$C_{sf} = \frac{(BW_c)}{2\pi f_o^2 C_s R_L} \quad (4.29)$$

$$L_{sf} = \frac{R_L L_s}{2\pi(BW_c)} \quad (4.30)$$

where f_o is the geometric center frequency of the filter given by

$$f_o = \sqrt{f_1 f_4} = \sqrt{f_2 f_3} \quad (4.31)$$

4.3.6 Design Procedure for a Band-Stop Filter

Similar to designing a bandpass filter, the following is a step-by-step procedure how to design a band-stop filter based on a low-pass prototype.

- a. Define or specify the response in terms of
 - Desirable attenuation at frequencies of interest
 - Maximum allowable ripple in the passband
 - Ratio of the source resistance R_S and the load resistance R_L
- b. Normalize the cut-off bandwidth BW_c in terms of the bandwidth of interest BW to determine the ratios BW_c/BW
- c. Determine the appropriate filter type and the minimum order to meet the required specification using either attenuation equations or curves for low-pass prototypes using the ratios BW_c/BW (as opposed to the ratios f/f_c)
- d. Determine the low-pass prototype values of the components
- e. Replace each inductor in the low-pass prototype with a parallel resonant circuit and each capacitor with a series resonant circuit as illustrated in Fig. 4.15. Note that both elements in each of the resonant circuits have the same normalized value.
- f. Scale all the elements to the desired frequency and impedance as follows.

For *series* resonant components,

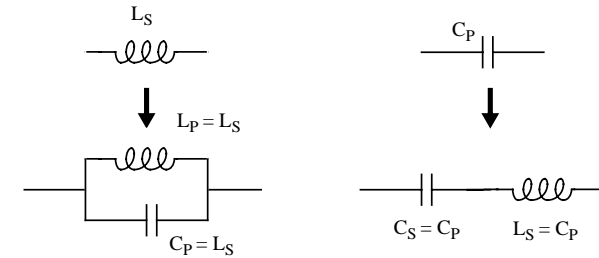


Fig. 4.15 Transformation of components from low-pass to band-stop filter

$$C_{sf} = \frac{C_p}{2\pi(BW_c)R_L} \quad (4.32)$$

$$L_{sf} = \frac{R_L(BW_c)}{2\pi f_o^2 L_p} \quad (4.33)$$

For *parallel* resonant components,

$$C_{pf} = \frac{(BW_c)}{2\pi f_o^2 C_s R_L} \quad (4.34)$$

$$L_{pf} = \frac{R_L L_s}{2\pi(BW_c)} \quad (4.35)$$

4.4 References

- [1] C. Bowick, *RF Circuit Design*, Indianapolis, Sams, 1982.
- [2] R. Gregorian, and G. Temes, *Analog MOS Integrated Circuits for Signal Processing*, New York, Wiley, 1986.
- [3] Y. P. Tsividis, and J. O. Voorman, *Integrated Continuous-Time Filters: Principles, Design, and Applications*, New York, IEEE Press, 1993.